Westminster School Challenge 2003

Mathematics III

1½ hours

You may not use a calculator in this paper.

All working should be clearly shown.

You should attempt as many questions as possible, in any order you like.

1 a I travel 104 kilometres in 1 hour and 20 minutes.
How long would it take me to travel 143 kilometres at the same speed?

b A journey takes me 2 hours and 24 minutes at 75 kilometres per hour.
How long would the same journey take at 100 kilometres per hour?

2 Adam, Bill and Carl share a packet of sweets. Between them, Bill and Carl eat $\frac{8}{15}$ of the sweets.

Between them, Adam and Bill eat $\frac{11}{12}$ of the sweets.

a What fraction of the sweets does Bill eat?

b If each boy eats a whole number of sweets, what is the smallest possible number of sweets in the packet?

3 a i Multiply out $(x+1)^2$.

ii Show how to use your result in part i to find 101^2.

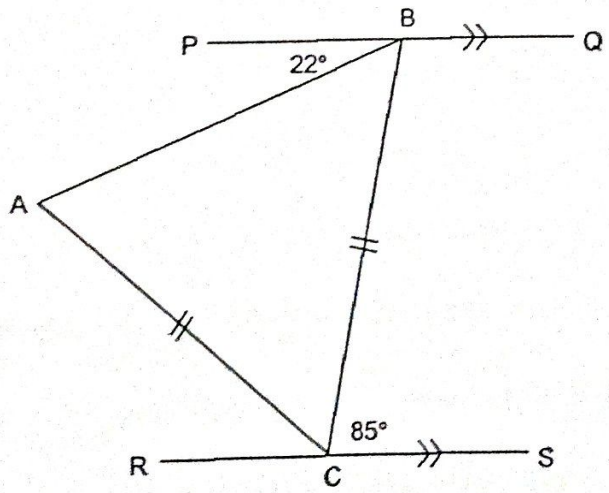
iii What is the least square number larger than one million?

b i Factorise x^2+8x+7

ii Show that 9000024000007 is not prime.

iii Give the prime factorisation of 1008007.

4 In the diagram, PQ is parallel to RS and AC and BC are equal in length.



Angle BCS = 85° and angle PBA = 22°.

Find angle ACR, giving and explaining each step in your working.

5 The Sixth Form at Westminster consists of 40% girls and 60% boys.
All of the girls and some of the boys work hard. Altogether, $\frac{2}{3}$ of the Sixth Form work hard.
What fraction of the boys work hard?

6 In a competition, Toby runs at 8 kilometres per hour for x kilometres and cycles at 20 kilometres per hour for the remainder of the 45 km course.

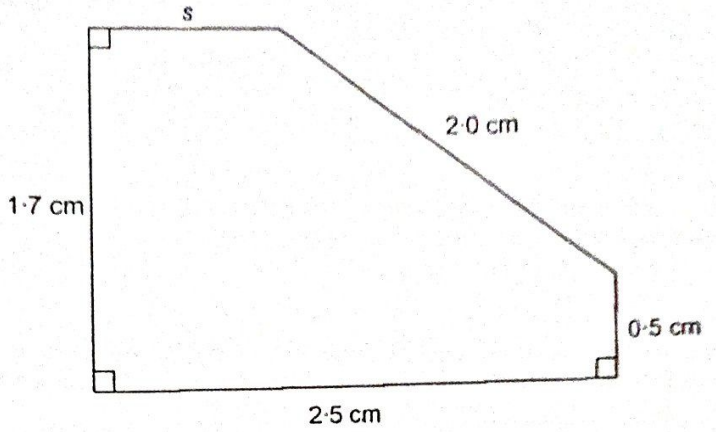
a Write an expression in terms of x for the time he takes.

Toby's average speed over the whole 45 kilometre course is 15 kilometres per hour.

b Write an equation for x and solve it to find the distance Toby ran.

7 The diagram shows an irregular pentagon. The lengths of four of the sides are shown in the diagram. Three of the angles in the pentagon are right angles, as shown.

Find the length of the side marked s.



8 a For the numbers $x = 2\frac{1}{3}$ and $y = \frac{7}{11}$,

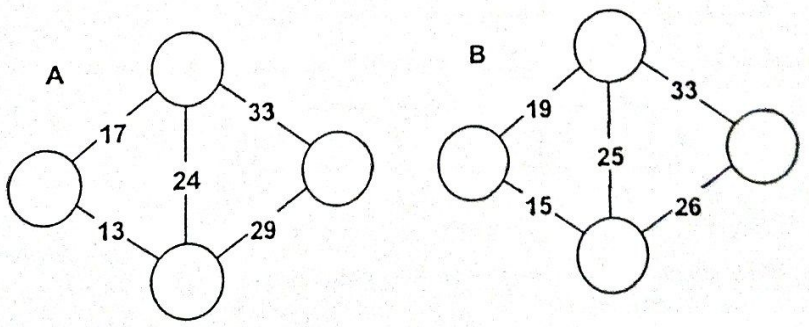
i find the product of x and y,

ii find the mean average of x and y,

and verify that these are equal.

b Find two other numbers whose product is equal to their mean average, and which are not equal to each other.

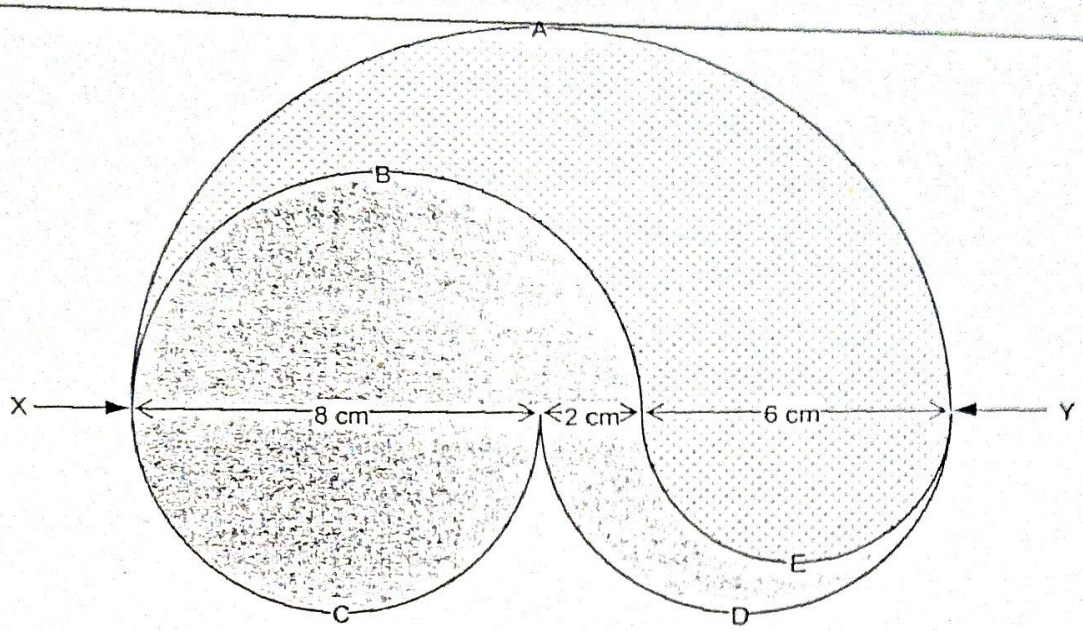
9 In the diagrams below, a number is to be written in each of the four circles, so that the two numbers in the circles at the ends of each line add up to the total given on the line.



a Find the numbers that can be written in each circle to solve problem **A**.

b Show that it is not possible to find numbers that can be written in each circle to solve problem **B**.

10 In the diagram, the figure shown is made up of 5 semicircles labelled A, B, C, D, E. All their diameters lie along the line XY.



a Show that the grey region and the dotted region have the same perimeter.

b Show that the grey region and the dotted region have the same area.

You may leave lengths and areas as multiples of π.

11 The game of boxes is played on the blank squares on the bottom row of the grid below. Players A and B take it in turns to write their letter in one of the blank squares, or in two *adjacent* blank squares.

1	2	3	4	5	6	7	8	9

If there are no blank squares for a player to fill in, then he loses.

a In the situation shown below, it is A's go. Where should A write his letter, or his two adjacent letters, in order to win? You should explain how A can be sure that he will win with the move you suggest.

1	2	3	4	5	6	7	8	9
		A		B	A	A	B	B

b In the situation shown below, it is B's go. Explain carefully why A can win, whatever B does.

1	2	3	4	5	6	7	8	9
		A			A	A	B	B

c If A has first go, which player ought to win and what should his strategy be?