Challenge 2017 Maths 3 solutions

1)   Ravi pays £x so $\frac{x}{24}$ per cookie.

        Sam pays £1.2x so $\frac{1.2x}{29}$ per cookie.

        Sam: $\frac{x}{24} \times \frac{1.2}{1.2} = \frac{1.2x}{28.8}$ so Sam gets better value.

2)   £24,500

3)   60

4)   2 hours

5)   Simultaneous equations lead to b=7
        5b+6t=65 and 6(11-b)+5(13-t)=64.

6)   Height of trapezium A = 12cm
        Height of trapezium B = 8cm

7)   s=12, p=3, m=5

8) a) $17n^2 + 30n + 17$
   b) $44^2 + 9^2$

9) a)   P=9.9, Q=13.3
   b)   R=11.75, S=15, T=18.25
   c)   U=2a-b, V=2b-a, W=3b-2a
   d)   nth term = -1+3n
   e)   nth term = 2a-b+n(b-a)

10)   h=3r

11)   8.5 litres of red paint and 11.5 litres of purple paint

12)a) i) Each pair adds to 2018 and there are 2017 pairs
          $\frac{2018 \times 2017}{2} = 2,035,153$
    ii)   1009
    iii) $\frac{1009 \times 2018}{2} = 1,018,081$
  b) i) $\frac{12 \times (6 \times 6)}{2} = 216$
    ii) $2n - 1$
    iii) $\frac{2n \times n^2}{2} = n^3$

13)a)i) One of $\frac{1}{a}$ and $\frac{1}{b}$ must be less than $\frac{1}{2}$ of $\frac{1}{4}$ and one must be more than $\frac{1}{2}$ of $\frac{1}{4}$.
          Therefore, one of a and b must be greater than 4 and one must be less than 4, respectively.

ii) (a,b) = (8,8), (6,12), (12,6), (5,20), (20,5)

b) i) Extending the shared straight line between the polygons into the square gives two exterior angles of size $\frac{360}{a}$ and $\frac{360}{b}$. These add to 90°.

ii) Divide the equation in part b)i) by 4 to get the equation in part a). Hence the solutions are (8,8), (6,12), (12,6), (5,20), (20,5).

14)a) Let, for example, pw mean the number of students who had pizza on Monday and a wrap on Tuesday.
If 12 students have pizza on one day and a sandwich on the other day then 26 students have wraps overall. The other numbers follow easily in the table (see below).
Number of students who have a sandwich on Monday = 15.

	S	P	W	
M	15 (sp+sw)	6 (ps+pw)	**17** (ws+wp)	**38**
T	**13** (ps+ws)	16 (sp+wp)	9 (sw+pw)	**38**
	28	**22**	26	**76**

b) i) So if pw = 0 then using the information in the table (expressions and numbers) ps = 6 (from Monday pizza entry in the table), ws = 7, wp = 10, sp = 6, sw = 9, which are all viable.

ii) From the table entry for Monday pizza we can see that ps + pw = 6.
So pw must be less than or equal to 6.
Check that pw = 6 works: wp = 4, ws = 13, ps = 0, sp = 12, sw = 3. So it works.