

Eton King's Scholarship 2007 B Solutions

1. a) 1/12  
b) 75 litres  
c) 28 and 40
2. a) 25 hours 36 minutes  
b) 3:4
3. 1980. Consider each number from 2 upwards. 2 must be right (as otherwise all multiples of 2 are also wrong and that is too many wrong, plus they aren't neighbouring). Similarly for 3 and 4. For 5, if 5 is wrong then so is 10 but they aren't neighbouring. 2 and 3 are correct so 6 is also. 7,8,9 we come back to later. 2 and 5 are correct so 10 is. 3 and 4 are correct so 12 is. 11 is sandwiched between 10 and 12 so must be correct. The LCM of all of those so far counted as good is 660 (take the highest occurring power of each prime factor that appears in any of the numbers). Now either 7 and 8 are the wrong ones (and we then must include 9 and multiply our LCM by 3, giving 1980), or 8 and 9 are the wrong ones (and we must include 7 and multiply our LCM by 7 giving 4620, which is too large).
4. a)  $x=1/8, y=1/3$   
b)  $p=3, q=1/2$
5. a) 275 degrees, 195 degrees  
b) Let  $DCE=BEC=y$  and  $ABC=EBC=x$   
 $BCA = 180-2y$  (angles at C add to 180 degrees)  
 $BEC = y-x$  (angles in triangle BCE add to 180 degrees)  
 $BAC = 2y-2x$  (angles in triangle BAC add to 180 degrees)  
Hence  $BAC = 2 \times BEC$
6. a)  $r^2 - 6r + 5 = 0$   
b)  $(r-1)(r-5)=0$  so  $r=5$   
c)  $r^2 - 8r + 10 = 0$  and substitute, complete the square or use quadratic formula.
7. a) ' $ab$ ' =  $10a+b$ , ' $ba$ ' =  $10b+a$  so the difference is  $|10a+b-(10b+a)| = |9a-9b| = 9|a-b|$ , which is a multiple of 9 and so not prime.  
b) It can't end (and so start) in 9 as 9 is odd. So make it end and start in 8, to maximise. Take the second and fifth digits as 9 to maximise. Now check third and fourth digits so that the digit sum is a multiple of 3, starting with 9 (to maximise) and working backwards. 7 is the first that works. Hence 897798.
8. a) Perimeter = area  
 $2x+2y = xy$   
 $xy-2x-2y+4=4$   
 $(x-2)(y-2)=4$   
 $(x-2,y-2)=(4,1)$  or  $(2,2)$ , so  $(x,y)=(6,3)$  or  $(4,4)$   
b)  $6x+6y=xy$ , which gives  $(x-6)(y-6)=6^2$

$x-6$	$y-6$	$x$	$y$
36	1	42	7
18	2	24	8
12	3	18	9
9	4	15	10
6	6	12	12

9. a) 1-9 is 9 digits, 10-99 is  $90 \times 2 = 180$  digits (to find the number of digits from 10-99 take 9 from each number to give 1-90) 100 is 3 digits. Overall 192 digits.  
 b) Number of pages from 101 is  $x-100$ . So number of digits is  $3(x-100)+192=3x-108$ .  
 c)  $3x-108=kx$  so  $k=1$  or 2. If  $k=1$  then  $x=54$  (not allowed as less than 100), if  $k=2$  then  $x=108$ , which works. 108 pages and 216 digits.
10. a) Eleven – see below.

10p's	5p's	2p's	1p's
1			
	2		
	1	2	1
	1	1	3
	1		5
		5	
		4	2
		3	4
		2	6
		1	8
			10

b)

20p's	10p's	5p's	2p's	1p's	Option number
1					1
	1	11 options for the other 10p (including a 10p coin), as seen in part a.			11
		4			1
		3	Successively two 2p's (one 1p), one 2p (rest 1p's), etc		3
		2	Successively five 2p's (rest 1p's), four 2p's (rest 1p's), etc.		6
		1	Successively seven 2p's (rest 1p's), six 2p's (rest 1p's), etc.		8
			Successively ten 2p's, eight 2p's and two 1p's, etc		11
Total					41

Hints:

- 2) For question 2a use a table with columns lorries, gravel and time. Gravel per lorry per hour is a fixed amount throughout:  $\frac{G}{L \times T}$ . Fix one quantity at each step and adjust the other two. When adjusting G either (but not both!) of L and T must be adjusted by the same multiple/divisor. When adjusting L and T multiply one and divide the other.  
For question 2b it's important to convert the ratios to actual amounts by using 3x:5x:4x for some x. Generally for ratio questions of this type that is a helpful trick.
- 5) When doing bearings, extending the north line also south can be useful (enabling alternate angles).