Eton College King's Scholarship 2010 B solutions

1) a) $\frac{13}{120}$
   b) $\frac{uxz}{vy}$
   c) $\frac{23}{36}rs$

2) a) Divide both sides by 1000
   b) 83.810205
   c) 67890
   d) 0.001
   e) 1

3) a) $a^2 - b^2$
   b) i) $(a+b)(a-b)$
   ii) $(ab+cd)(ab-cd)$
   c) i) Second equation:
   $(2x+y)(2x-y) = 31$
   $31(2x-y) = 31$
   $2x - y = 1$ (equation 1)
   ii) $2x + y = 31$ (equation 2)
   Equation 2 – equation 1 gives $2y = 30$ so $y = 15$ and $x = 8$
   d) $9x^2 - 16y^2 = 176$
   $(3x+4y)(3x-4y) = 176$
   $44(3x-4y) = 176$
   $3x - 4y = 4$ (equation 3)
   $3x + 4y = 44$ from the question (equation 4)
   $8y = 40$ (equation 4 – equation 3)
   $y = 5$ and $x = 8$

4) a) $x = -1$
   b) i) $x - 8 = 5(y-8)$
   $x + 10 = 2(y+10)$
   ii) $x = 38, y = 14$

5) a) Angles at centre are $180 - 2p$ and similar.
   They all add to 360: $720 - 2(p+q+x+y) = 360$, hence result.
   b) i) $180 - (x+y)$
   ii) From a), BAD = $180 - (x+y)$
   From b), BCT = $180 - (x+y)$
   Therefore BAD = BCT
   c) From b)ii) XYV = middle angle marked
   From b)ii) middle angle marked = right-hand angle marked
   So XYV = right-hand angle marked.
   By corresponding angles XY and UV are parallel.

6) a) $2\pi R^2$

b) $4\pi R$
c) $45°$

7) a) 120cm
b) i) $(20 - x)^2 = 15^2 + x^2$
$x = \frac{35}{8}$
ii) CB=AD, DAQ=PAB (so all angles same) so congruent triangles and BP=DQ.
iii) $(20 - 2x)^2 + 15^2 = PQ^2$
iv) $\left(\frac{45}{4}\right)^2 + 15^2 = \left(\frac{75}{4}\right)^2$
$45^2 + 225 \times 16 = 75^2$, which works

8) a) 81
b) i) $1200 = 2^4 \times 3 \times 5^2 \ and \ 2880 = 2^6 \times 3^2 \times 5$
ii) 250

9) a) i) Perimeter = 2x + 2y
Diagonal = $\sqrt{x^2 + y^2}$
ii) $20 = 2(x + y) \ so \ 10 = x + y$
$8 = \sqrt{x^2 + y^2} \ so \ 64 = x^2 + y^2$
iii) $18cm^2$
b) i) 5cm
ii) $AD^2 = AC^2 + CD^2$
$AD = \sqrt{29}cm$
c) $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
d) The perimeter of a cuboid is the sum of the edges.
$320cm^2$

10) a) AM=CM, both of which are bases to triangles ABM and CBM respectively.
The height of both those triangles is the same. So the areas are the same.
Similarly BN=NA, both of which area bases to triangles CAN and CBN respectively.
The height of both those triangles is the same. So the areas are the same.
b) i) A1+A2=A3+A4
A1+A3=A2+A4
ii) Subtract the equations above:
A2-A3=A3-A2
2A2=2A3
A2=A3
c) Area AGM = A3 (=x), Area AGN = A2 (=x) so A1=2x, A4 = 2x and area ABC = 6x.
So A4 is 1/3 of ABC.
d) $\frac{3}{4}$