# Eton College King's Scholarship Examination 2018

# MATHEMATICS B

**(One and a half hours)**

*Candidate Number:...........................................................................*

**<u>Please write your candidate number on EVERY sheet.</u>**

**Please answer on the paper in the spaces provided.**

*Each question is worth 10 marks.*

*Show all your working.*

*Answers without sufficient working may receive little or no credit.*

**The use of calculators is permittted.**

# Do not turn over until told to do so.

1. a) Solve for *x:*

$$\frac{1}{3 + \frac{1}{x}} = \frac{3}{16}$$

b) Solve for *x:*

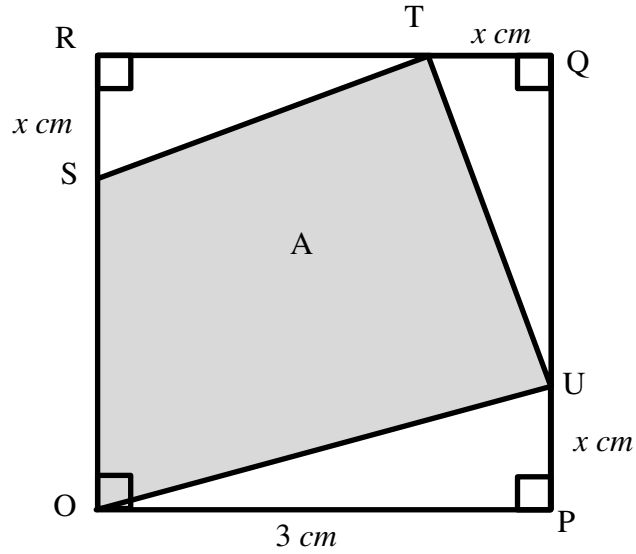$$\frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} = \frac{5}{7}$$

Notice that

$$\frac{5}{11} = \frac{1}{11/5} = \frac{1}{2 + \frac{1}{5}}$$

c) Find positive integers $a$, $b$, and $c$ such that

$$\cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c}}} = \frac{7}{17}$$

2.

R          T    *x cm*
Q

*x cm*

S

A

U

*x cm*

O                P
3 cm

OPQR is a square of side length 3 cm. PU = QT = RS = $x$ cm, where $0 < x < 3$. The shaded region OSTU has area A.

a) Show that $A = x^2 - \frac{9}{2}x + 9$

b) Expand $\left(x - \frac{9}{4}\right)^2$, simplifying fully.

c) Hence find a number $d$ such that $A = \left(x - \frac{9}{4}\right)^2 + d$

d) For what value of $x$ is $A$ smallest, and what is this area?

3. Parallel lines A and B are 120m apart on flat ground. Jeff and Will both run back and forth between line A and line B, following parallel straight lines, without bumping into each other.

A                                                                                                                                    B



120m

Jeff starts at line A. When a whistle is blown at time $t = 0$, he runs towards line B at 4m/s. When he reaches line B he instantaneously turns and runs at the exact same speed back towards line A. He then instantaneously turns and returns to line B, and so on, indefinitely.

Will starts at line B. When the whistle is blown at time $t = 0$, he runs towards line A at 6m/s, and continues to run back and forth in the same way as Jeff.

a)    What is the first time that Jeff and Will pass by each other?

b)    How far from line B are Jeff and Will when they first pass each other?

c)    What is the second time that Jeff and Will pass each other?

d)    How far from line A are Jeff and Will when they pass each other for the third time?

4. a) Show that $(1 + x + x^2)(1 - x) = 1 - x^3$

b) *Without* using a calculator and showing full working, use part a) to show that:
$$1 + \frac{2}{7} + \frac{4}{49} = \frac{1 - \frac{8}{343}}{1 - \frac{2}{7}}$$

c) Multiply out $(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8)(1 - x)$ and simplify fully:

d) *Without* using a calculator and showing full working, evaluate the following sum, leaving your answer as a fraction:
$$\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \frac{64}{729} + \frac{128}{2187} + \frac{256}{6561}$$

5. Suppose $x = 0.1\dot{2}\dot{3} = 0.1232323 \ldots$

a) State the value of $10x$

b) State the value of $1000x$

c) Hence show that $990x = 122$ and write $x$ as a fraction in its lowest terms.

Suppose now that $y = 17.\dot{a}\dot{b} = 17.ababab \ldots$where $a$ and $b$ are integers between 0 and 9.

d) By considering $100y$, show that $y = \frac{1683+10a+b}{99}$

e) If $z = 71.7\dot{c}\dot{d}$, find integers $m$ and $n$ such that $z = \frac{m+10c+d}{n}$

6.a) Express $\frac{1}{x} + \frac{1}{y}$ as a single fraction.

b) Show $x^2 + y^2 = (x + y)^2 - 2xy$

If $xy = 5$ and $x + y = 8$:

c) Calculate the value of $\frac{1}{x^2} + \frac{1}{y^2}$, leaving your answer as a fraction.

d) By considering $(x + y)^3$, calculate the value of $\frac{1}{x^3} + \frac{1}{y^3}$, leaving your answer as a fraction.

7. *Throughout this question, leave all answers as exact expressions in terms of $\pi$ and square roots, for example $2(\pi + \sqrt{3})$ or $\frac{2\pi}{\sqrt{5}}$ etc.*

In the diagram below, the circle centred at O has radius 3cm. The curved line AOC is part of a circle, also of radius 3cm, centred at D. OD is perpendicular to AC.
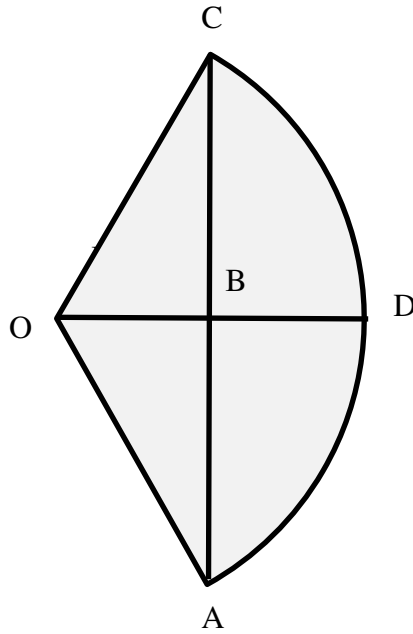
### DIAGRAM 1
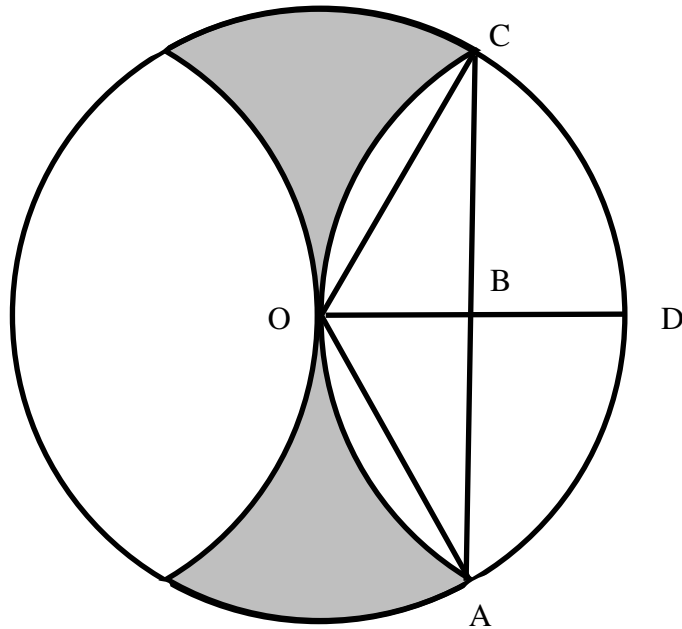


a) Calculate the length AB.

b) Calculate the area of triangle AOC.

c) Calculate the area of sector OADC in diagram 1 (indicated for reference in diagram 2 below).

**DIAGRAM 2**

C

B

O

D

A

d) Calculate the shaded area indicated in **diagram 1**.

8. Sarah has an accurate analogue wristwatch with a standard 12-hour face including hour and minute hands.
What is the smaller angle between the minute and hour hands when the time is:

a) 19:45?

b) 15:27?

c) What is the first time after 3pm that the angle between the two hands is 119°?

d) Sarah starts her homework shortly after 5pm, when the angle between the hands is 128°. Her homework takes her more than 1 hour. When she finishes the homework, the angle between the hands is 12°. What is the smallest possible time it took Sarah to complete her homework?

END OF PAPER