Westminster School Challenge 2006

Mathematics II

1½ hours

You are expected to use a calculator in this paper.

All working should be clearly shown.

You should attempt all the questions, in any order you like.

1 Robert watches Dr Who videos without any break between them. Each episode lasts 52 minutes. If Robert has 11 hours to spare,

a how many episodes can he watch completely;

b how many minutes of the last episode he starts watching does he miss?

2 Robin hires a car at 8:00 a.m. The company charges him 6 pence per minute for hiring the car. They also charge 92 pence per litre for the petrol he uses. He sets off immediately and travels to a friend's house, which is 143 kilometres away, at an average speed of 55 kilometres per hour. He spends an hour and three quarters at his friends house, and then drives back to return the car, at an average speed of 65 kilometres per hour.

a At what time does he return the car?

The company charges Robin a total of £30·25 for car hire and petrol.

b How may litres of petrol does he use?

3 **a** Simplify $x(1-x)-x-1$.

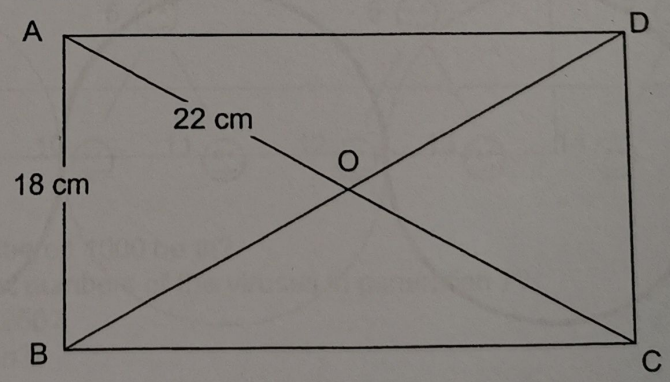
b What is the result of multiplying $\frac{1}{2x}+\frac{2}{x}$ by $2x$?

c Solve the equation $\sqrt{x+1}=3$.

d Solve the simultaneous equations

$$y-2x=2$$
$$11x+2y=1$$

4 In rectangle ABCD, AO = 22 cm and AB = 18 cm.
Find the area of the rectangle.



5 **a** A car which normally costs £14, 400 is reduced to £12, 600.
What percentage reduction is this?

b The same percentage reduction is applied to a more expensive car.
This car is reduced to £27, 300. What does this car normally cost?

6 Alex takes 24 minutes to jog around the park. Bertie takes 36 minutes to jog around the park. Chris takes 54 minutes to jog around the park. The three boys start together at midday at the bandstand in the park and jog around and around the park, maintaining a constant speed. They keep going until the first time at which all three again reach the bandstand simultaneously.

a i At what time does this occur?

ii How many times has each boy jogged round the park at this time?

The band plays the Star Wars theme endlessly, without a break. They start playing at midday, and *each* time *any* of the boys passes the bandstand, they are just starting to play the theme again.

b How long might the Star Wars theme take to play once?

7 The number 85 is equal to $9^2 + 2^2$ and equal to $7^2 + 6^2$. So 85 can be written as the sum of two exact square numbers in two different ways.

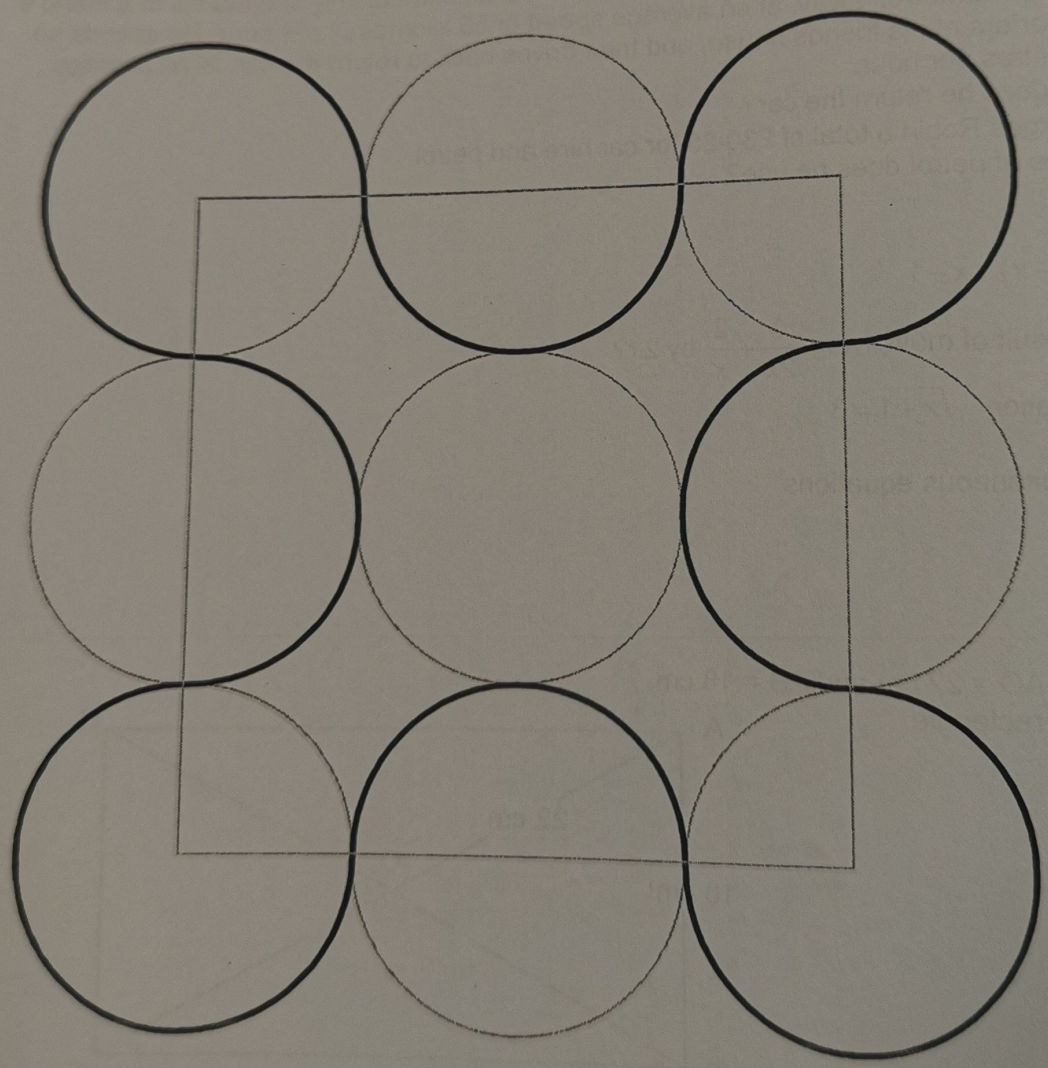
The number 1105 can be written as the sum of two exact square numbers in four different ways.

Use your calculator to find the four different ways of writing 1105 as the sum of two exact square numbers.

8 The diagram shows nine circles, and a square which passes through the centres of eight of the circles.

The length of the perimeter of the figure outlined with a heavy black line is 40 cm.

What is the radius of one of the circles?



9 A school expedition is rather badly organised. There are ***N*** children signed up for the trip. One fifth of them are left behind at school. Eight are mislaid at a service station on the way. Half the remainder are lost on the expedition. Five more are mislaid at a service station on the way back.

a Write an expression in ***N*** for the number of children who make it back to school.

In fact, one quarter of the children who signed up for the trip make it back to school.

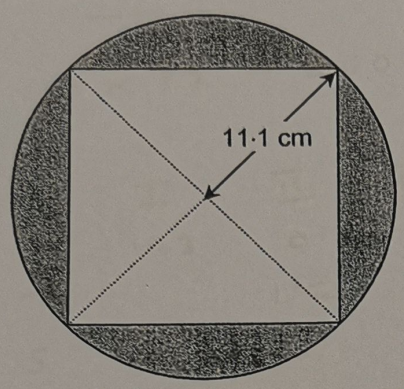
b Find ***N***.

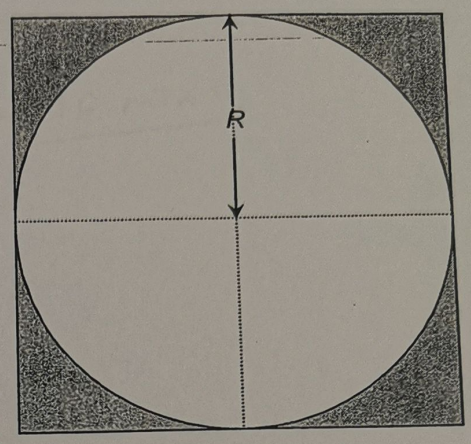
10 **a** The population of mayflies on a loch rises by 5% per day every day for a fortnight (14 days). What is the overall percentage increase in the population of mayflies over this period?

b What daily rise in the population of locusts in an oasis for a week (7 days) would lead to an overall increase in the population of 100% over this period?

11 The first diagram shows a square whose vertices lie on the circumference of a circle. The circle has radius 11·1 cm.

a What is the shaded area?

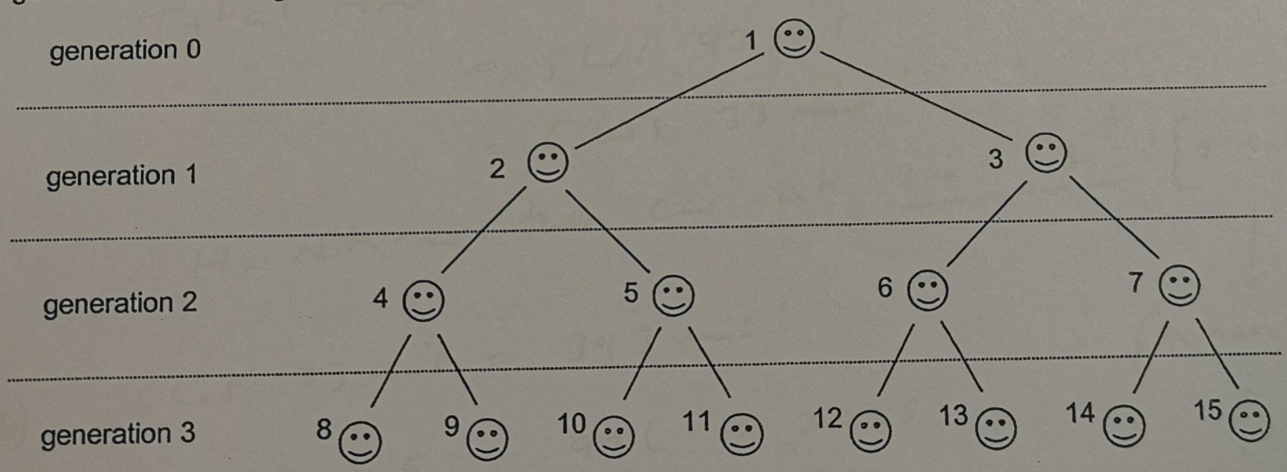




The second diagram shows a circle which just fits inside a square.
The shaded area in the second diagram is exactly equal to the shaded area in the first diagram.

b What is the radius R of the second circle?

12 A computer virus works by making two copies of itself. The code for each copy of the virus contains a unique number which identifies the copy. The original is copy 1; it is generation 0. The copies it makes are numbered 2 and 3. These two copies are called generation 1. The two copies made by copy 2 are numbered 4 and 5; the two copies made by copy 3 are numbered 6 and 7. These four copies are generation 2. The diagram below shows this information, and generation 3.



a Which generation will the virus numbered 1000 be in?

b What will be the smallest and largest numbers of the viruses in generation 7?

c What are the copies made by copy 250?

d What are the copies made by copy n?

e What copy made copy 4013?

f Describe a general method for finding which copy made copy n.

Each copy has a chain of *ancestors*: the copy that made it, the copy that made the copy that made it, and so on back to copy 1.

g Find a copy with number larger than 2006 that has all its ancestors odd numbered.

h Give a general rule for the numbers of the copies that have all their ancestors odd numbered.

i *Prove* that copies which have numbers obeying your general rule have all their ancestors odd numbered.