Eton College King's Scholarship 2003 A solutions

1) a) 18cm
   b) 1.25 Australian dollars
   c) x = 9, y = 12
   d) x=12cm, y=13cm
   e) $21p^3 + 55p^2 + 4pq - 21q^2$
   f) $x = d - \frac{b}{(a-f)^2}$
   g) i) $\frac{2}{5}$
      ii) $\frac{11}{36}$
   h) 25,25,27,56,100

2 a) COA = 180-2x
   COB = 180-2y
   $x + y = 20°$
   b) If BAT = p then BAO = 90-p
   OAB is isosceles so AOB = 2p
   Similarly to part a, do angles around O add to 360. So (180-2x)+(180-2y)+2p=360
   Which gives x+y=p
   c) $60°$

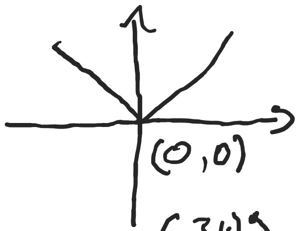
3) a) 5050
   b) $\frac{n(n+1)}{2}$
   c) Series should read 2+5+8+…+**2996+2999**+3002
   1,503,502
   d) i) a+9d
      ii) a+(n-1)d
      iii) $n\ times\ the\ average\ of\ the\ first\ and\ last\ terms$
      $n \times \frac{((a)+(a+(n-1)d))}{2}$
      $\frac{n}{2}(2a + (n-1)d)$

4) a) i) PQ = 4
      ii) Area OPQ = 4
      iii) OM = 2
      iv) Area of shaded segment = $2\pi - 4$
   b) $3\pi - 2\sqrt{2}$

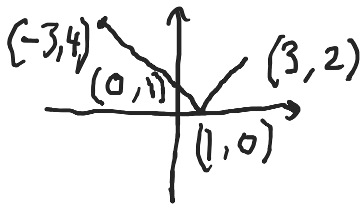
5) a) $t^4 + 2t^2 + 1$
   b) $(2t)^2 + (t^2 - 1)^2$
   $= 4t^2 + t^4 - 2t^2 + 1$
   $= t^4 + 2t^2 + 1$
   $= (t^2 + 1)^2$, as required
   c) ABC = 90 degrees by Pythagoras
   d) o x o = o so $t^2 \pm 1$ is even and 2t even.
   e) One of $t^2 - 1, t^2, t^2 + 1$ is divisible by 3 as they are consecutive integers.

If $t^2 - 1$ or $t^2 + 1$ are divisible by 3 then we have our result.
If it is t^2 that is divisible by 3, then t is divisible by 3 and also 2t, hence result.

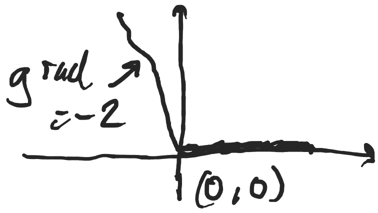
6) a)



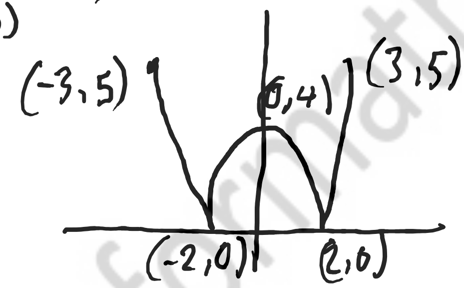
b)



c)



d)



7)

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  P Q R S
+ T U V Q
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T U R Q W
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T = 1 as the highest carry-over is 1

Consider P + T + possible carry-over = P+1 or P+2.
We need a carry-over so P=8 or 9, giving U=0 or 1.
But 1 is already taken.
So **U = 0.**

P = 8 or 9.
If P = 8 then there must have been a carry-over from (Q + 0(U) + (necessary) carry-over from R+V). But then Q must be 9 giving Q + 0(U) + (necessary) carry-over = 0(R).
However, R can't be 0 as U=0.
So **P = 9** and Q + 0(U) + possible carry-over is less than 10.

R + V + possible carry-over must give a carry-over so that Q and R aren't the same.
Q+1=R.

R + V + possible carry-over ends in Q.
Q + 1 + V + possible carry-over ends in Q
Q + 1 + V + possible carry-over = 10 + Q
1 + V + possible carry-over = 10 so V = 8 or 9.
V can't be 9 (already taken) so **V=8** and there is a carry-over from S+Q.

S and Q can be a maximum now of 7+6 = 13 and a minimum of 10.
So W=2 or 3 (0 and 1 are taken).
If S and Q are 6/7, or 7/6, then as R= Q+1 then R must be 7 or 8.
But these are taken.
So S and Q are 5 and 7 and **W=2**.

R = Q+1 so **Q=5** (R cannot be 8, which is taken), **R=6** and **S=7**.