Eton King's Scholarship 2012 Maths A solutions

1) a) i) $\frac{7}{9}$

ii) $-1\frac{37}{60}$

b) i) $x < 30$

ii) $x > 2$

c) Yes: 55,55,70 and 65,65,50

d) i) £6

ii) £98.79

e) 0.6cm

f) i) $-13\frac{2}{3}$

ii) $x = -11$

g) $107°$

h) x=3, y=-1

i) i) $27a^3b^{12}$

ii) $\frac{1}{4}$

j) i) $\sqrt{1125} = 15\sqrt{5} \approx 33.5m^2$

ii) $35.4m^2$

k) i) $\frac{5}{16}$

ii) $\frac{2b}{3a}$

l) i) £26,080,000

ii) £12,990

iii) 250

2) a) Join the centres of the circles to make a square side length $2\sqrt{2}$. This has diagonal length 4 by Pythagoras. So the diameter = $4 + 2\sqrt{2}$. So the radius is $2 + \sqrt{2}$

b) Large circle area = $\pi\left(2 + \sqrt{2}\right)^2 = \pi\left(6 + 4\sqrt{2}\right)$

4 x small circle area = $4 \times 2\pi = 8\pi$

Difference = $\pi\left(4\sqrt{2} - 2\right) = 2\pi\left(\sqrt{2} - 1\right)$

c) Circumference of large circle + 4 x (circumference of small circle)

$= 2\pi\left(2 + \sqrt{2}\right) + 4 \times 2\pi \times \sqrt{2}$

$= 4\pi + 2\sqrt{2}\pi + 8\sqrt{2}\pi$

$4\pi + 10\sqrt{2}\pi$

3) a) 1,45,3,15,5,9

b) All the factors of 45 are odd, so the sum of any two of them is even

c) The numbers are either 1,32 (sum 33) or 2,16 (sum 18) or 4,8 (sum 12). The only odd sum is 33.

d) The numbers are either 1,81 (sum 82) or 3,27 (sum 30) or 9,9 (sum 18).

e) 4 and 354,294

4) a) i) 1.8cm

ii) 8cm

b) Easy expansion

c) PQ=$(c-x)^2$

QS=$\sqrt{a^2-(c-x)^2}$

x =$\sqrt{b^2-(a^2-(c-x)^2)}$

x =$\sqrt{b^2-a^2+c^2-2cx+x^2}$

$x^2=b^2-a^2+c^2-2cx+x^2$

$x=\frac{b^2+c^2-a^2}{2c}$

5) a) 10 letters and 9 addresses so at least one of the addresses must be duplicated

b) $\frac{230}{7}=32\frac{6}{7}$. So there are 32 birthdays on each day then there are still Etonians left over. So there must be at least one day with more than 32 Etonians having a birthday on that day.

c) The minimal situation is 1+2+3+4+5+6+7+8=36. So it is not possible.

d) Divide the rectangle into 3x3cm squares. At best, the first 8 points are in different squares. The remaining point must now share a square with another point. The maximum distance between them will be $3\sqrt{2}$.

6) a) Easy expansion and rearrange.

b) 10,13,23,29,35 has mean 22.
10+35-22=23.
So the set becomes 13,22,23,23,29 after one application.
This has mean 22 and 13+29-22=20.
So the set becomes 20,22,22,23,23

c) Applying again: the mean is 22 and 20+23-22=21 so the set becomes 21,22,22,22,23.
Applying again: the mean is 22 and 21+23-22=22 so the set becomes 22,22,22,22,22.

d) The product of the numbers is only changed because the largest and smallest numbers (x and y) changed to A and x+y-A. Now x<=A and y>=A so (x-A)(y-A)<=0, so xy<=A(x+y-A). Thus the change described results in a product bigger or equal to the original product.

e) The R algorithm changes {x,y} to {A,x+y-A}. But the sum of these two pairs is just x+y, so the mean, A, is unchanged.

f) By part d), and c) the product 10x13x23x29x35<=22^5