Eton 2017 King's Scholarship A Solutions

1) a) i) £8640

ii) £1400

b) i) $x = \frac{2}{3}$

ii) $x = \frac{5}{7}$

c) $\frac{15}{31}$

d) $420cm^2$

e) $x = 3, y = -2$

f) i) $x > 2$

ii) $x < -6$

g) 16.25

h) i) 360

ii) 80

i) i) $\frac{3}{2xy}$

ii) $\frac{3x^2}{2y}$

j) i) Subtract 100×473.

80883

ii) 128,311,183

k) Question error: answer is $90 + \frac{x}{2}$

n sides: exterior angle is $180 - x$

2n sides: exterior angle is $\frac{180-x}{2}$

2n sides: interior angle is $180 - \frac{180-x}{2} = 90 + \frac{x}{2}$

2) a) Easy

b) There are 4 places where the up step can occur

c) 6 (U first, then three options for other U; or R first, then three options for other R)

d) U then 4 options for the other U (as in part b).

Or R then 6 options (as in part c).

e) U then 5 options for other U (similarly to part b)

Or R then follow part d

Gives 15 overall.

3) a) $\pi(r + 1) + 2r + 2 = \pi r + 2r + 2h$

$h = \frac{\pi+2}{2}$

b) $1\frac{1}{10}cm$

4) a) ABC = 75 (isosceles)

CBD = 15 (90-75)

BDE = 150 (BDE isosceles and angles add to 180)

ADB = 15 (180-90-75)

ADE = 135 (BDE – ADB)

b) Extend PR by a length equal to PQ.

This gives a total bottom length PT (T is the new point) which is the same as QS.

PQ=QR=RT=TS.

Let QPR = x

Then QST = x (opposite angles in parallelogram)
RTS = 180-x (allied angles)
TSR = x/2 (triangle TSR isosceles and angles in triangle add to 180)
QSR = QST – TSR = x/2, as required.

5) a) i) "ab" = 10a +b
"ba" = 10b+a
So the sum = 11(a+b)

ii) 11(a+b) = 143 when a+b=13 so "ab" = 49, 58, 67, 76, 85, 94, which is six numbers

b) i) "abcd" = 1000a+100b+10c+d
"dcba" = 1000d +100c+10b+a
So the sum is 1001(a+d) + 110(b+c) = 11(91(a+d)+10(b+c)), so a multiple of 11

ii) 1001(a+d) + 110(b+c)
= 143(7(a+d)) + 110(b+c).
The first half of the above expression is already a multiple of 143.
The second half needs either to be zero (b+c=0) or otherwise is a multiple of 143 if 13 is a factor of b+c, which is only possible if b+c=13.

6) a) Same diagram as bottom row but with an additional 16 squares on the right.

b) The patterns demonstrate that the sum of powers of two from 2^0 to 2^n add to $2^{n+1} - 1$.

c) {1} is sum-free
{1,2} is sum-free
{1,2,4} is sum-free as the above is sum free, and is larger than the sum of the above elements.
At each stage we include a further power of 2. The existing powers of 2 (including 1) are sum-free, and the new power of 2 is larger than the sum of the existing elements.