Eton 2018 King's Scholarship B Solutions

1) a) $x = \frac{3}{7}$

b) $x = \frac{2}{3}$

c) $\frac{1}{2+\frac{1}{2+\frac{1}{3}}}$

2) a) $A = 9 - 2\left(\frac{1}{2}\left(x(3-x)\right)\right) - \frac{3x}{2}$

$A = 9 - x(3-x) - \frac{3x}{2}$

$A = x^2 - \frac{9}{2}x + 9$

b) $x^2 - \frac{9}{2}x + \frac{81}{16}$

c) $d = 9 - \frac{81}{16} = \frac{63}{16}$

d) $x = \frac{9}{4}, A = \frac{63}{16}$

3) a) 12s

b) 72m

c) 36s

d) 96m

4) a) $1 + x + x^2 - x - x^2 - x^3 = 1 - x^3$

b) $\left(1 + \frac{2}{7} + \left(\frac{2}{7}\right)^2\right)\left(1 - \frac{2}{7}\right) = 1 - \left(\frac{2}{7}\right)^3 = 1 - \frac{8}{343}, as\ required.$

c) $1 - x^9$

d) $1\frac{6049}{6561}$

5) a) $1.\dot{2}\dot{3}$

b) $123.\dot{2}\dot{3}$

c) $1000x - 10x = 123.\dot{2}\dot{3} - 1.\dot{2}\dot{3} = 122\ so\ 990x = 122\ and\ x = \frac{61}{495}$

d) $100y = 17ab.\dot{a}\dot{b} = 1700 + 10a + b$

$y = 17.\dot{a}\dot{b}$

$99y = 17ab - 17 = 1700 + 10a + b - 17 = 1683 + 10a + b, hence\ result$

e) $1000z - 10z = 717cd - 717 = 71700 - 717 + 10c + d$

$z = \frac{70983+10c+d}{990}$

6) a) $\frac{x+y}{xy}$

b) $(x+y)^2 - 2xy = x^2 + 2xy + y^2 - 2xy = x^2 + y^2$

c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2+y^2}{x^2y^2} = \frac{(x+y)^2-2xy}{x^2\,y^2} = \frac{64-10}{25} = \frac{54}{25}$

d) $\frac{1}{x^3} + \frac{1}{y^3} = \frac{x^3+y^3}{x^3y^3} = \frac{(x+y)^3-3x^2y-3xy^2}{x^3y^3} = \frac{(x+y)^3-3xy(x+y)}{x^3y^3} = \frac{(8^3-3\times5\times8)}{125} = \frac{392}{125}$

7) a) $\frac{3\sqrt{3}}{2}$

b) $\frac{9\sqrt{3}}{4}$

c) 3π

d) $9\sqrt{3} - 3\pi$

8) a) $37.5°$

b) $58.5°$

c) 15:38
d) 1 hour 26 minutes and 33 seconds