

## Eton 2020 King's Scholarship B Solutions

- 1) a)  $225\text{cm}^2$   
b) i)  $1 : 1.33$   
c)  $240\text{cm}^2$
- 2) a)  $38.5\text{kg}$   
b)  $4.6\text{kg}$
- 3) a) Let  $\angle BCA = x$ . Then  $\angle BAC = 90 - x$  and  $\angle ABD = x$ . And have all the angles needed.  
b)  $6.72\text{cm}$   
c) Left square side length  $x$ . Lengths along bottom:  $\frac{4}{3}x + x = 4$ .  $x = \frac{12}{7}$   
Right square side length  $y$ . Lengths along bottom:  $\frac{4}{3}y + y + \frac{3}{4}y = 5$ .  $y = \frac{60}{37}$ .  
Ratio follows.
- 4) a) A must be Teacher as the others say they are something else so couldn't be the truthful Teacher.  
B can't be Oppidan as then telling truth. So B is the Scholar. C is the Oppidan.  
b) A can't be the Oppidan as then would be telling the truth. A lying so Scholar.  
B must be lying so B is the Oppidan.  
C Teacher.  
c) C can't be Teacher as then would say I am teacher. B is telling truth. B is Scholar. If B were teacher then would agree with C. C Oppidan. A Teacher.  
d) If C telling truth then A Oppidan then B Teacher and C Scholar.  
If C lying then A not Oppidan (so A is Teacher or Scholar) then A could be lying or not depending on whether Teacher or Scholar.  
If A Teacher then B is Scholar and C is Oppidan.  
If A is Scholar then B is Teacher and C is Oppidan.  
So not enough information to determine who Teacher is and B lying.  
So A Teacher, B Scholar and C Oppidan.
- 5) a)  $p = -4\frac{1}{2}, q = 17\frac{1}{2}, r = 19\frac{1}{2}$   
b)  $x = 31.25, y = 36, z = 24$   
c)  $a = 6, b = 8, c = 12$
- 6) a) 37 and 73  
b)  $x=5, y=7, z=8$
- 7) a) i)  $4 < 5\frac{7}{13} < 6 < 6\frac{1}{2} < 9$   
ii)  $50 < 94\frac{2}{17} < 200 < 425 < 800$   
b) i) Any square number is  $\geq 0$  so  $(a - b)^2 \geq 0$   
ii)  $(a - b)^2 \geq 0$   
 $a^2 - 2ab + b^2 \geq 0$   
 $2ab \leq a^2 + b^2$   
c) Let  $a = \sqrt{x}$  and  $b = \sqrt{y}$  and substitute in previous part, then dividing by 2.  
d) In part c, divide both sides by  $(x + y)$  and multiply both sides by 2 and  $\sqrt{xy}$

- e) For  $x, y$  positive, parts b to d (following the equivalence backwards) show that  $\frac{2xy}{x+y} = \sqrt{xy}$  (i.e., HM=GM) occurs when  $(\sqrt{x} - \sqrt{y})^2 = 0$ , which occurs when  $x = y$ . In this case each mean =  $x$ . For  $x$  and  $y$  of different signs, the GM does not exist. For  $x$  and  $y$  both negative, the AM and GM both exist and are equal ( $x$ ) but the harmonic mean is  $-x$ .

8) a) Robin is at the start line after 10, 20, 30 minutes. After 10 and 20 the others aren't at the start, after 30 they are.

b) After 15 minutes Robin is halfway round (1.5 laps) and Joel (12 laps) and Nick (5 laps) are at the start line.

c) Speeds are  $\frac{4}{5}, \frac{1}{3}, \frac{1}{10}$  laps per minute.

The distance between Joel and Nick opens by half a lap every  $s$  minutes, where

$$\frac{4}{5}s - \frac{1}{3}s = \frac{1}{2} \text{ so } s = \frac{15}{14}$$

The distance between Nick and Robin opens by half a lap every  $t$  minutes, where

$$\frac{1}{3}t - \frac{1}{10}t = \frac{1}{2} \text{ so } t = \frac{15}{7}$$

The distance between Joel and Robin opens by half a lap every  $u$  minutes, where

$$\frac{4}{5}u - \frac{1}{10}u = \frac{1}{2} \text{ so } u = \frac{5}{7}$$

So after  $\frac{15}{7}$  minutes (taking the longest of those times and noticing that the other two divide into  $\frac{15}{7}$ ) they are all on a straight line.

Joel has done  $\frac{4}{5} \times \frac{15}{7} = \frac{12}{7} = 1\frac{5}{7}$  laps.

Nick has done  $\frac{1}{3} \times \frac{15}{7} = \frac{5}{7}$  laps.

Robin has done  $\frac{1}{10} \times \frac{15}{7} = \frac{3}{14}$  laps.

So Joel and Nick are both opposite Robin at that point.

The time is  $\frac{15}{7}$  minutes  $\approx 2$  minutes and 9 seconds