Eton 2021 King's Scholarship A Solutions

1) a) $28\frac{1}{3}$

   b) $18\frac{2}{3}$

   c) $599\frac{5}{36}$

   d) $10\frac{9}{16}$

2) a) 0.000077

   b) 7.5

   c) -1.331

3) a) $1\frac{1}{2}$

   b) -5

4) a) $8x - 15y$

   b) a

5) $x > -3\frac{1}{5}$

6) $3\frac{11}{20}$

7) 16 years

8) $113°$

9) 65.2%

10) 6

11) a) 2 hours 55 minutes

    b) $\frac{AB}{A+B}$

    c) $\frac{3pr^2}{5q^2}$

    d) $\frac{1\frac{7}{15}yz}{x}$

12) a) 2:25

    b) $2\pi rh$

    c) $29\pi p^2$

13) a) i) $10x = 75.\dot{3}$

       $100x = 753.\dot{3}$

    ii) The difference between the equations above gives: $90x = 678$.

       Divide both sides by 90 gives $x = \frac{678}{90}$ $i.e.\ 7.5\dot{3} = 7\frac{8}{15}$

    b) $y = 1.\dot{9}$

       $10y = 19.\dot{9}$

       $9y = 18$

       $y = 2$

    c) $10z = 17b.\dot{c}$

       $100z = 17bc.\dot{c}$

       $90z = 17bc - 17b = 1000 + 700 + 10b + c - 100 - 70 - b = 1530 + 9b + c$

       $z = \frac{1530 + 9b + c}{90}$

14) a) Triangle AMC is right angled with AM=MC=$\sqrt{2}$ so by Pythagoras AC=2

    b) By symmetry on DM, angle MLC=$90°$. Similarly MNC=$90°$. So NML=$90°$.

       So CNML is a rectangle (and perhaps a square).

CL=CN=1 by symmetry. So LM=MN=1 also (opposite sides in a rectangle).

c) ML=1 (as above). MD=$\sqrt{2}$. So DL=$\sqrt{2}-1$

d) DE=$\sqrt{1^2+\left(\sqrt{2}-1\right)^2}=\sqrt{4-2\sqrt{2}}$

e) Arc AE = $\sqrt{2}\pi > AB + BC + CD + DE = 4DE$

$$\sqrt{2}\pi > 4\sqrt{4-2\sqrt{2}}$$

$$2\pi > 4\sqrt{2}\sqrt{4-2\sqrt{2}}$$

$$2\pi > 4\sqrt{2\left(4-2\sqrt{2}\right)}$$

$$\pi > 2\sqrt{8-4\sqrt{2}}$$

$$\pi > 2\sqrt{4\left(2-\sqrt{2}\right)}$$

$$\pi > 4\sqrt{2-\sqrt{2}}$$

15)a)

Stage	Pole A Lowest position → Highest		Pole B Lowest position → Highest		Pole C Lowest position → Highest	
Initial Position	2	1				
Move One	2		1			
Move Two			1		2	
Move Three					2	1

b)

Stage	Pole A Lowest position → Highest			Pole B Lowest position → Highest			Pole C Lowest position → Highest		
Initial Position	3	2	1						
Move One	3	2					1		
Move Two	3			2			1		
Move Three	3			2	1				
Move Four				2	1		3		
Move Five	1			2			3		
Move Six	1						3	2	
Move Seven							3	2	1

c)

Stage	Pole A Lowest position → Highest				Pole B Lowest position → Highest				Pole C Lowest position → Highest			
Initial Position	4	3	2	1								
Move One	4	3	2		1							
Move Two	4	3			1				2			
Move Three	4	3							2	1		
Move Four	4				3				2	1		
Move Five	4	1			3				2			
Move Six	4	1			3	2						
Move Seven	4				3	2	1					
Move Eight					3	2	1		4			
Move Nine					3	2			4	1		
Move Ten	2				3				4	1		
Move Eleven	2	1			3				4			
Move Twelve	2	1							4	3		
Move Thirteen	2				1				4	3		
Move Fourteen					1				4	3	2	
Move Fifteen									4	3	2	1

d) $2^n - 1$. When we have an extra disc, we have to shift all those above ('small pile') to another pole ($2^{n-1} - 1$), then shift the bottom disc and then shift the 'small pile' back on top of the biggest disc ($2^{n-1} - 1$).

$(2^{n-1} - 1) + (2^{n-1} - 1) + 1 = 2^n - 1$