Eton College King's Scholarship Examination 2022

MATHEMATICS A

(One and a half hours)

Candidate Number:……

Please write your candidate number on EVERY sheet.

Please answer on the paper in the spaces provided.

This paper is divided into two sections:
Section I (Short-answer questions) – 50 marks available
Section II (Extended questions) – 50 marks available

Answer all of Section I and as many questions as you can from Section II.

The marks for each part of each question are given in square brackets.

Show all your working.

No diagram is drawn to scale.

Neither calculators nor protractors may be used.

***ADDITIONAL MATERIALS:** NONE*

Do not turn over until told to do so.

Section I: Short-answer questions (50 marks)

1. Fully simplify the following expressions.
   a. $5y + 16 - 8y - 4$

[1]

   b. $\frac{3}{4}y - \frac{y}{2}$

[1]

   c. $\frac{y \times y \times y \times y \times y}{y^3}$

[1]

2. Find the value of the following, giving your answers as **reduced, mixed fractions.**

a. $115\frac{1}{3} + 62\frac{2}{5} - 71\frac{4}{9}$

[3]

b. $\left(\frac{81}{98} \div 3\frac{6}{7}\right) \times \frac{154}{9}$

[4]

c. $\left(3\frac{2}{3} - 2\right)^2$

[3]

3. Find the value of the following, giving your answers as **a decimal**.

   a. 0.018×0.0045

[3]

   b. $0.2 - (0.2)^3$

[4]

   c. $0.403 \div 0.062$

[3]

4.

2	15	60
12	47	9
51	19	34

I choose a number at random from the grid shown above. I am equally likely to choose any of these numbers. Giving your answers as **fully simplified fractions,** find the probability that the number I choose is:

a. odd;

[1]

b. a multiple of 5;

[1]

c. a prime number;

[1]

d. not a multiple of 3.

[1]

5. If $p = 6$, $q = -2$ and $r = \frac{1}{2}$, find the value of the following expressions, **fully simplifying your answers.**

   a. $p^2 - \frac{2q}{r}$

[3]

   b. $\frac{p^3+q^3}{p+q}$

[3]

6. Solve the following simultaneous equations.

$$5x + 7y = 11$$
$$4x + 3y = 14$$

[4]

7. Solve the following equation for x. Give your answer as a **reduced, mixed fraction.**

$$\frac{2x + 1}{4} + \frac{x - 1}{10} = 2$$

[3]

8. There are 8 more boys than girls in a football squad of 32. What is the ratio of girls to boys in the squad in simplified form?

[3]

9. Annie is 44 years old and her father is 36 years older than her. How many years ago was Annie's father three times her age?

[3]

10. Megan, Flossy and Emma are sisters. Megan is 80% taller than Emma. Flossy is 25% taller than Emma. Find the percentage by which Megan is taller than Flossy.

[4]

Section II: Extended questions (50 marks)

11.

a. I visit three stores with some pocket money. I spend one fifth of my pocket money in the first shop and one quarter of what remains in the second shop. What fraction of the original amount is left over to spend in the third shop?

[3]

b. I am choosing a new washing machine. The standard model costs £200 and uses $17\frac{1}{2}$ pence of electricity per hour in operation. The energy-saving model costs £550 but only uses $3\frac{1}{2}$ pence of electricity per hour in operation. I operate my washing machine for twenty hours a week, on average. If I buy the energy-saving model, after how many weeks would my reduced electricity costs balance out the additional purchase price?

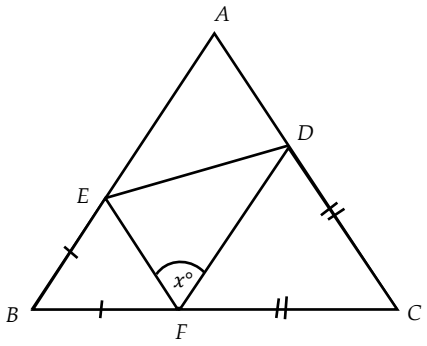
[3]

c. I run a cake stall on a Friday afternoon and spend 10 hours making 60 cakes. The ingredients for each cake costs me £1.60 and I charge £6 an hour for my labour. The sale price for a cake is 75% more than the total cost of making it. I have a special offer on this week which gives 20% discount on my normal sale price. How much will a customer pay for one of my cakes this week?

[4]

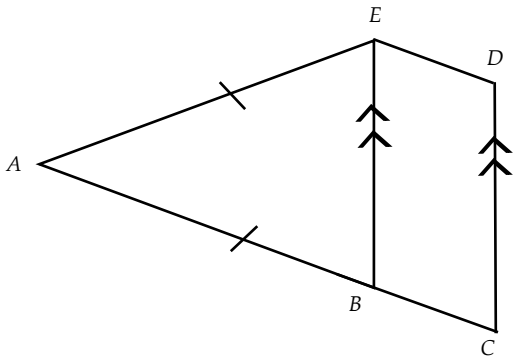
12.

a. A triangle ABC is made up of four smaller triangles. EBF and DFC are isosceles triangles. Angle EFD is $x°$ and is acute. Calculate the size of the angle EAD in terms of x.



[3]

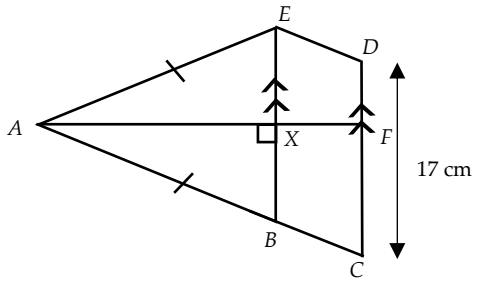
b. The shape below is formed by joining an isosceles triangle ABE and a parallelogram $BCDE$. ABC is a straight line and angle BCD is $\left(\frac{90-5y}{4}\right)^{\circ}$.



i. Find angle BAE in terms of y, **fully simplifying your answer.**

[3]

The line AF intersects BE at X as shown in the diagram below. It is given that the length $DC = 17$ cm and the ratio of the length AX and XF is $3:2$.

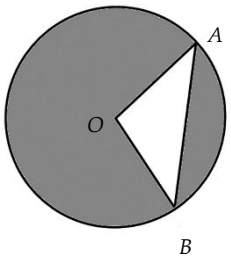


ii. Given that the area of ABE is 78 cm^2, calculate the area of the parallelogram $BCDE$.

[4]

13.

a. A triangle OAB is cut out of a circle with centre O. OA and OB are radii of length 35 cm and the chord AB is of length 56 cm.



i. Find the area of the triangle OAB.

[2]

ii. Using $\frac{22}{7}$ as an approximation for π, calculate an estimate for the shaded area, **to the nearest 10 square centimetres.**

[2]

b. Pentagon $ABCDE$ is irregular but all five sides are each 34 cm long. Diagonal AC is 66 cm and is parallel to side ED.

   i. Show that the area of triangle ABC is $33\sqrt{x}$ cm^2, where x is a positive whole number less than 100.

[2]

ii. Show, by calculating its value, that the area of quadrilateral $ACDE$ is a whole number of square centimetres.

[3]

iii. By making a suitable estimate for $\sqrt{x}$ in part i., find the area of pentagon $ABCDE$ **correct to the nearest 10 square centimetres.**

[1]

14.

a. If $m = \frac{7+n}{2}$ and $n = \frac{1+m}{2}$, find the value of $\frac{m+n}{2}$.

[2]

b. x, y and z are positive numbers.

$$x \times y = 6$$
$$y \times z = 27$$
$$z \times x = 2$$

i. Find the value of $(xyz)^2$.

[2]

ii. Hence find the values of x, y and z.

[3]

c.

$$u + v = -3$$
$$v + w = -2$$
$$w + u = 6$$

Find the values of u, v and w.

[3]

15.

a. X is a three-digit whole number. The sum of its digits is 12. If the second and third digits (that is, the "tens" digit and the "units" digit) are switched, the resulting number is 45 more than X.

Showing your method clearly, find all possible values for X.

[Hint: let the letters a, b and c represent the three digits of X and start by finding two equations involving a, b and c].

[4]

b. The numbers 1327231 and 394493 are both *palindromic*: they read the same when the order of their digits is reversed.

i. Find the largest five-digit palindromic whole number which is divisible by 3 but not divisible by 9.

[2]

ii. Of all possible five-digit palindromic whole numbers divisible by 15, Y is the largest and Z is the smallest. Find the difference between Y and Z.

[4]

END OF PAPER