

Sixth Form Entrance 2015

MATHEMATICS

1 hour

Attempt all questions if possible. Do not worry if there are topics you have never covered; do your best on whatever you can attempt.

Questions are not necessarily in order of difficulty.

Marks for parts of questions are given in brackets as a guide.

Show as much working as you can. Calculators are allowed and their use expected.

There is a list of formulae at the front, not all of which need necessarily be used in this paper.

The paper has twenty-four questions. Work quickly.

There are one hundred and fifteen marks in total.

NAME: **AGE:**

PRESENT SCHOOL:

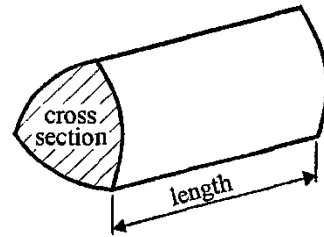
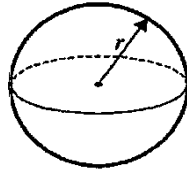
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Formula Sheet

Volume of prism = area of cross-section \times length

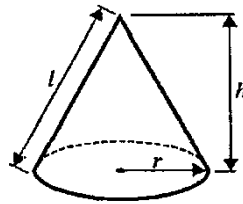
$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$

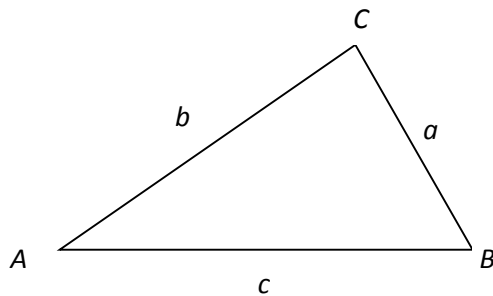


$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area of cone} = \pi r l$$



In any triangle ABC



Sine Rule:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule:
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of a triangle} = \frac{1}{2} ab \sin C$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q1. Karen buys 19 identical calculators. The total cost is £143.64

Work out the total cost of 31 of these calculators.

£

(Total 3 marks)

Q2. There are 40 litres of water in a barrel.

The water flows out of the barrel at a rate of 125 millilitres per second.

1 litre = 1000 millilitres.

Work out the time it takes for the barrel to empty completely.

..... seconds

(Total 3 marks)

Q3. In a sale all the normal prices are reduced by 18%.
In the sale Mandy pays £12.71 for a hat.

Calculate the normal price of the hat.

£.....

(Total 3 marks)

Q4. There are some sweets in a bag.

18 of the sweets are toffees.

12 of the sweets are mints.

(a) Write down the ratio of the number of toffees to the number of mints.
Give your ratio in its simplest form.

..... :

(2)

There are some oranges and apples in a box.

The total number of oranges and apples is 54

The ratio of the number of oranges to the number of apples is 1 : 5

(b) Work out the number of apples in the box.

.....

(2)

(Total 4 marks)

Q5. (a) Expand and simplify $(x - 17)(x - 4)$

..... (2)

(b) Solve $\frac{x - 17}{x - 4} = 5$

..... (3)

(Total 5 marks)

Q6. Solve the simultaneous equations

$$\begin{aligned} 2x + 3y &= 6 \\ 3x - 2y &= 22 \end{aligned}$$

$x =$

$y =$

(Total 4 marks)

Q7. Simplify fully

(i) $m^4 \times m^5$

.....

(ii) $p^6 \div p^2$

.....

(iii) $5x^3y \times 2xy^8$

.....

(iv) $\frac{4(k+8)^2}{k+8}$

.....

(Total 6 marks)

Q8. A straight line has equation $y = 4x - 5$.

Write down the equation of the straight line that is parallel to $y = 4x - 5$ and passes through the point (0, 3).

.....

(Total 2 marks)

Q9. (a) Work out, giving your answers as **integers** or **fractions** as appropriate:

(i) 8^0

(ii) 5^{-2}

(iii) $25^{\frac{1}{2}}$

(iv) $27^{-\frac{1}{3}}$

.....

.....

.....

.....

(7)

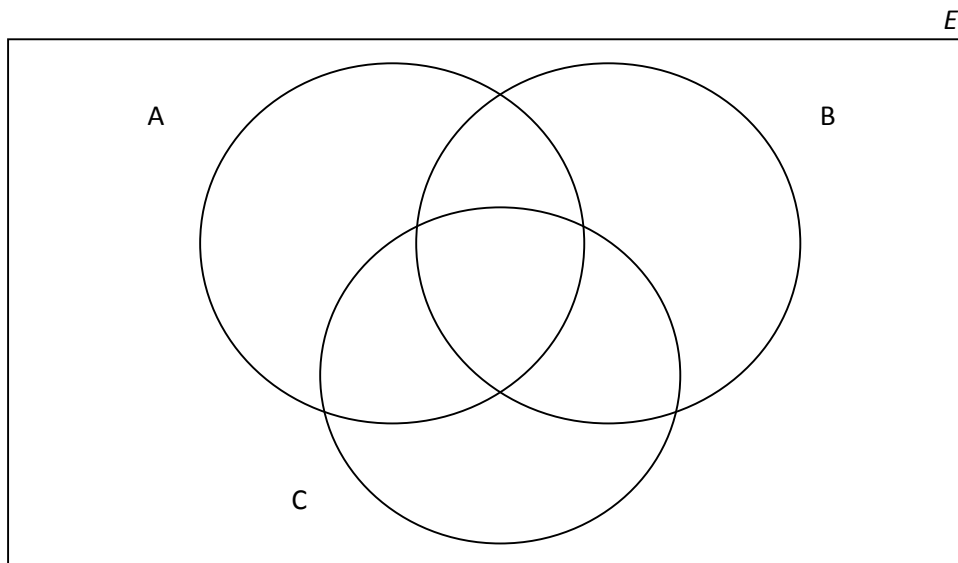
(b) Given that $x = 2^k$ and $\sqrt{\frac{4}{x}} = 2^c$, find c in terms of k .

$c = \dots\dots\dots$

(3)

(Total 10 marks)

Q10. The diagram shows three intersecting sets A, B and C.



a) Lightly shade the region described by $(A \cap B) \cap C'$

(1)

In a school all of the 120 pupils must choose at least one of Archery, Badminton or Cycling.

- 63 pupils choose archery
- 62 pupils choose badminton
- 69 pupils choose cycling
- 28 pupils choose archery and badminton
- 27 pupils choose archery and cycling
- 32 pupils choose badminton and cycling.

b) Let the number of people who choose all three options be x

i) Write down in terms of x the number of people who choose archery and badminton but not cycling.

.....**(1)**

ii) Show that the number of people who choose archery only is $8 + x$

.....**(2)**

iii) Form and solve an equation to find x .

.....**(3)**

(Total 7 marks)

Q11.

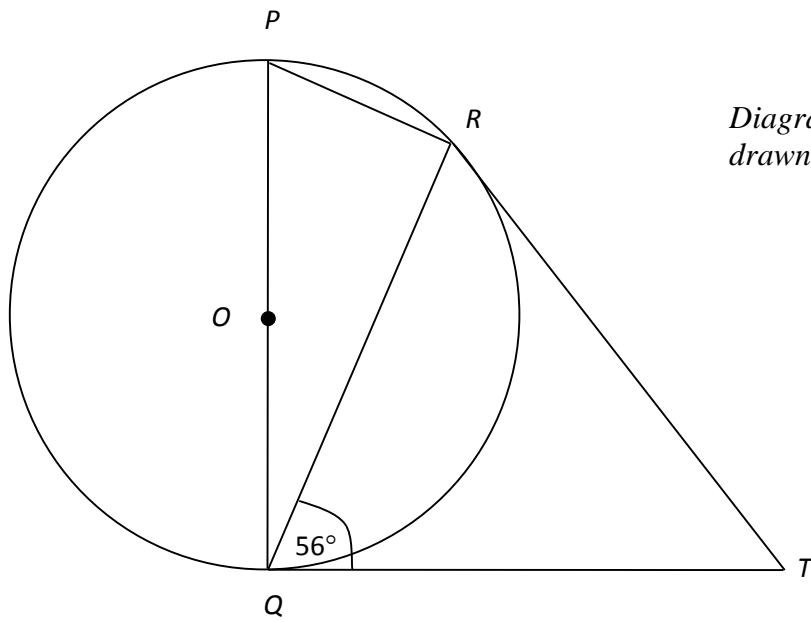


Diagram **NOT** accurately drawn

P , Q and R are points on a circle, centre O . POQ is a straight line.

TQ and TR are tangents to the circle.

Angle $TQR = 56^\circ$.

(a) Explain why angle $PQR = 34^\circ$.

.....

.....

(1)

(b) Calculate the size of angle PRT .
Give reasons for your answer.

.....^o
(3)

(Total 4 marks)

Q12. y is directly proportional to the **square** of x .

When $x = 4$, $y = 25$.

(a) Find an expression for y in terms of x .

(b) Calculate y when $x = 2$.

(c) Calculate x when $y = 9$.

.....
(3)

.....
(1)

.....
(2)

(Total 6 marks)

Q13.

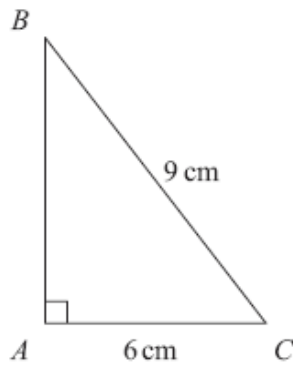


Diagram **NOT**
accurately drawn

ABC is a right-angled triangle.

$AC = 6$ cm.

$BC = 9$ cm.

Work out the length of AB . Give your answer correct to 3 significant figures.

..... cm
(Total 3 marks)

Q14. Sethina recorded the times, in minutes, taken to repair 80 car tyres.
Information about these times is shown in the table.

Time(t minutes)	Frequency		
$0 < t \leq 6$	15		
$6 < t \leq 12$	25		
$12 < t \leq 18$	20		
$18 < t \leq 24$	12		
$24 < t \leq 30$	8		

Calculate an estimate for the mean time taken to repair each car tyre.

..... minutes
(Total 4 marks)

Q15. Consider the formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Given $u = 2\frac{1}{2}$, $v = 3\frac{1}{3}$

(a) Find the value of f **without a calculator and showing working**

.....
(3)

(b) Rearrange $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

to make u the subject of the formula.

Give your answer in its simplest form.

.....
(3)
(Total 6 marks)

Q16. Here is a right-angled triangle.

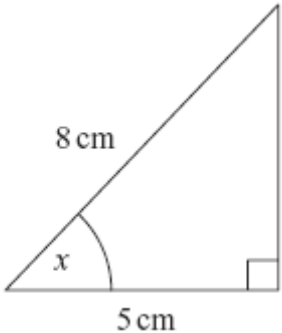


Diagram NOT accurately drawn

- (a) Calculate the size of the angle marked x .
Give your answer correct to 1 decimal place.

$x = \dots\dots\dots$
(3)

Here is another right-angled triangle.

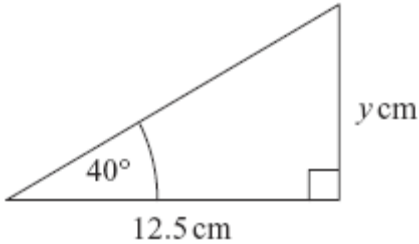


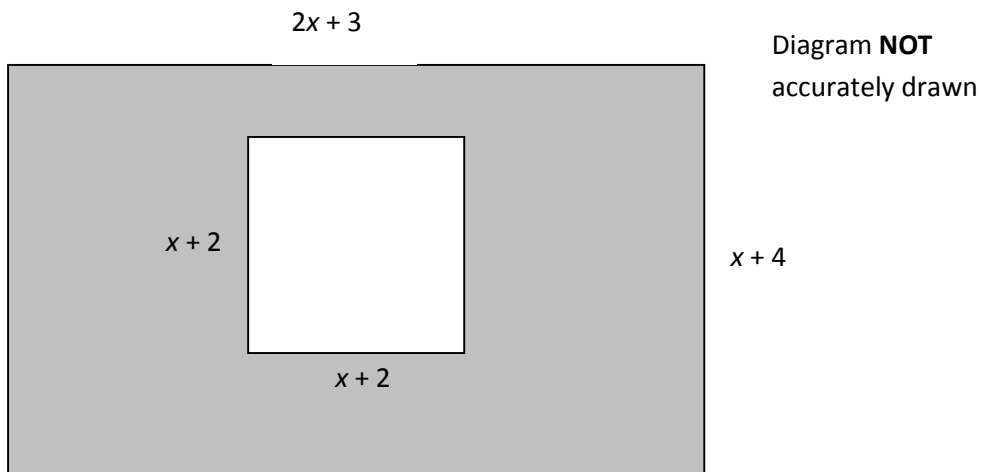
Diagram NOT accurately drawn

- (b) Calculate the value of y .
Give your answer correct to 1 decimal place.

$y = \dots\dots\dots$
(3)

(Total 6 marks)

Q17. Peter cuts a square out of a rectangular piece of metal.



The length of the rectangle is $2x + 3$.

The width of the rectangle is $x + 4$.

The length of the side of the square is $x + 2$.

All measurements are in centimetres.

The shaded shape in the diagram shows the metal remaining.

The area of the shaded shape is 20 cm^2 .

(a) Show that $x^2 + 7x - 12 = 0$

..... (4)

- (b) (i) Solve the equation $x^2 + 7x - 12 = 0$
Give your answers correct to 4 significant figures.

.....
(3)

- (ii) Hence find the perimeter of the **square**.
Give your answer correct to 3 significant figures.

..... cm
(1)

(Total 8 marks)

Q18. Phil has 20 sweets in a bag.

5 of the sweets are orange.

7 of the sweets are red.

8 of the sweets are yellow.

Phil takes at random **two** sweets from the bag.

Work out the probability that the sweets will **not** be the same colour.

.....

(Total 4 marks)

Q19. Simplify fully $\frac{x^2 - 8x + 15}{2x^2 - 7x - 15}$

.....

(Total 4 marks)

- Q20.** In the circle shown below, O is the centre and RV is a tangent touching the circle at point T. Angle $ATR = 62^\circ$ and angle $BTV = 79^\circ$. Find the value of the angle y , giving a **reason** for each step of your working.

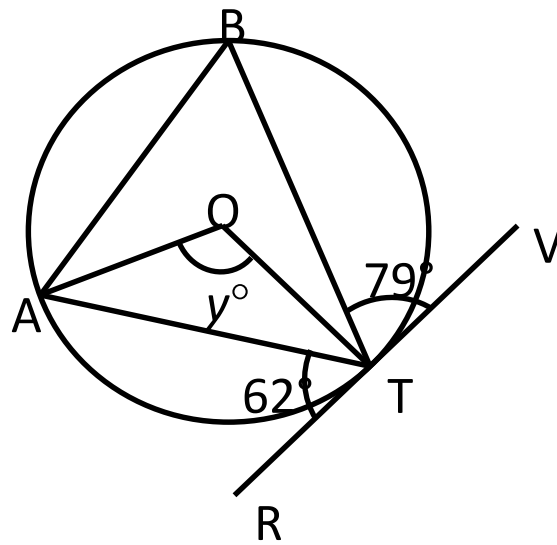


Diagram NOT
accurately drawn

$y = \dots\dots\dots^\circ$

(Total 4 marks)

Q21. Solve simultaneously the equations

$$y = x^2 - 3x + 7 \text{ and}$$

$$y = 2x + 1$$

.....
(Total 5 marks)

Q22.

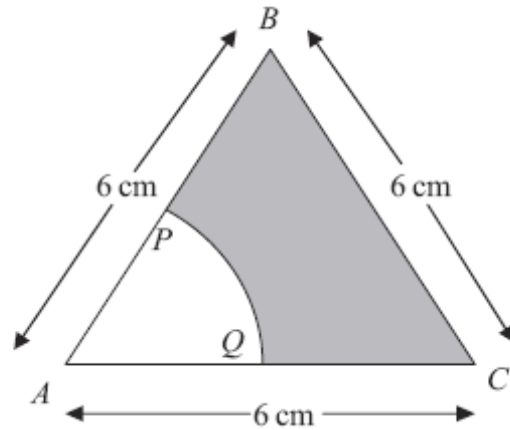


Diagram **NOT** accurately drawn

The diagram shows an equilateral triangle ABC with sides of length 6 cm.

P is the midpoint of AB .

Q is the midpoint of AC .

APQ is a sector of a circle, centre A .

Calculate the area of the shaded region.

Give your answer correct to 3 significant figures.

..... cm²

(Total 4 marks)

Q23. If $PQ = 7$ km, $PR = 11$ km and angle $QPR = 83^\circ$, work out length QR correct to 3 significant figures.

.....
(Total 5 marks)

Q24. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n .

(Total 5 marks)

END OF EXAM

<https://metalrockeducation.co.uk/>