

Sixth Form Entrance 2018

MATHEMATICS

1 hour

Write in this booklet.

Attempt all questions if possible. Do not worry if there are topics you have never covered; do your best on whatever you can attempt.

Questions are not necessarily in order of difficulty.

Marks for parts of questions are given in brackets as a guide.

Show as much working as you can. Calculators are allowed and their use expected.

There is a list of formulae given, not all of which need necessarily be used in this paper.

The paper has twenty-seven questions. Work quickly.

There are one hundred marks in total.

NAME: AGE:

PRESENT SCHOOL:

Q1.

The equation of the line L_1 is

$$y = 3x - 2$$

The equation of the line L_2 is

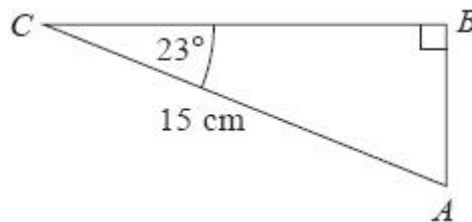
$$3y - 9x + 5 = 0$$

Show that these two lines are parallel.

(Total for question = 2 marks)

Q2.

ABC is a right-angled triangle.



Calculate the length of AB .

Give your answer correct to 3 significant figures.

..... cm

(Total for question = 2 marks)

Q3.

Julia buys a washing machine.

20% VAT is added to the price of the washing machine.

Julia then has to pay a total of £600

What is the price of the washing machine with **no** VAT added?

£

(Total for question = 2 marks)

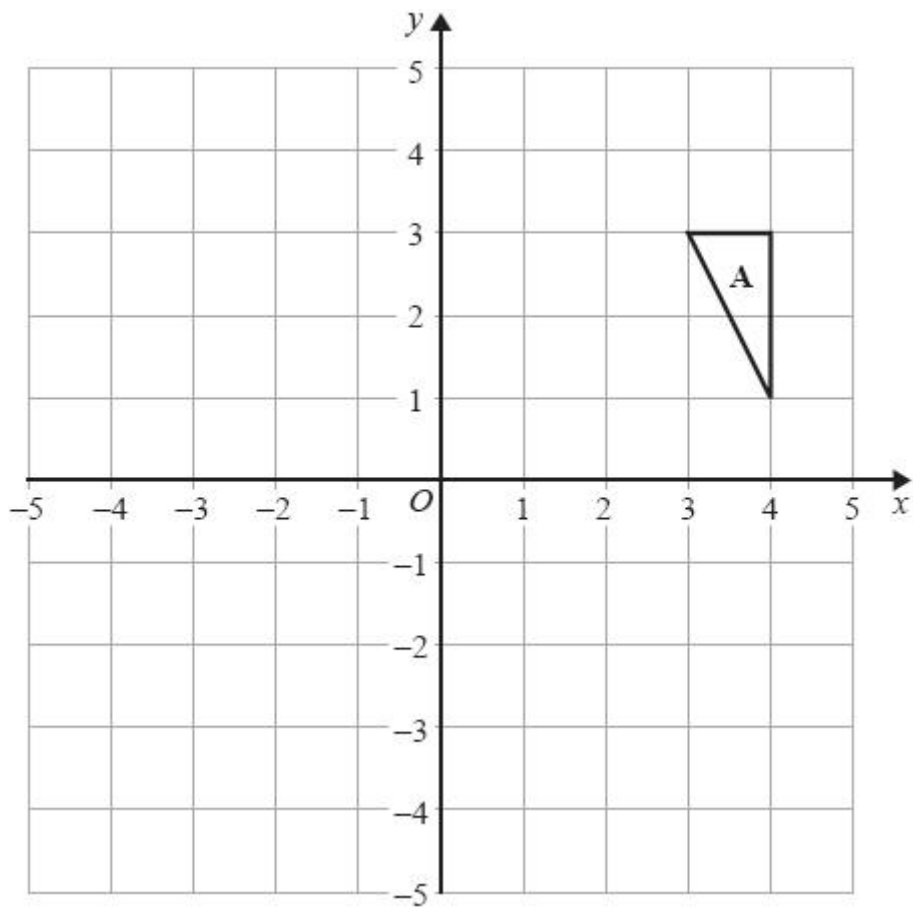
Q4.

Show that $(x + 1)(x + 2)(x + 3)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are positive integers.

(Total for question = 2 marks)

Q5.

The diagram shows triangle **A** drawn on a grid.



Rob reflects triangle **A** in the x -axis to get triangle **B**.
He then reflects triangle **B** in the line $y = x$ to get triangle **C**.

Sue reflects triangle **A** in the line $y = x$ to get triangle **D**.
She is then going to reflect triangle **D** in the x -axis to get triangle **E**.

Sue says that triangle **E** should be in the same position as triangle **C**.

Is Sue correct?
You must show how you get your answer.

(Total for question = 3 marks)

Q6.

Here are the first five terms of a sequence.

4 11 22 37 56

Find an expression, in terms of n , for the n th term of this sequence.

.....
(Total for question = 3 marks)

Q7.

There are 10 boys and 20 girls in a class.
The class has a test.

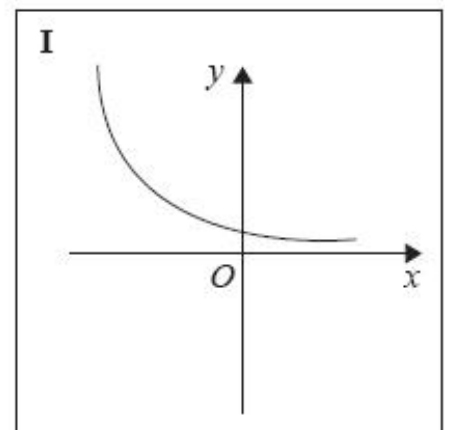
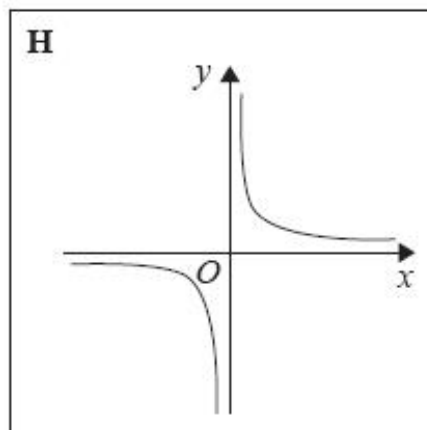
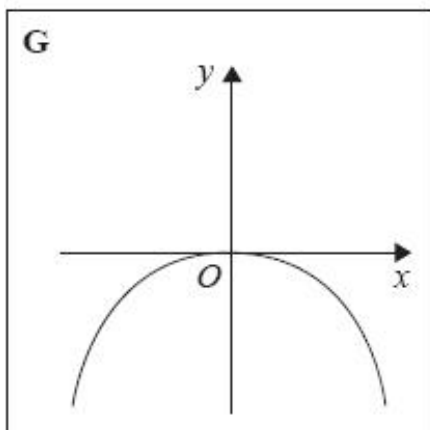
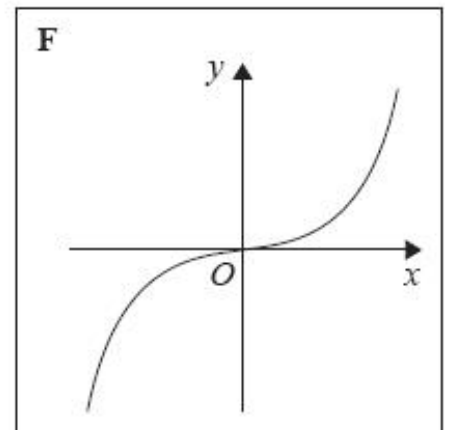
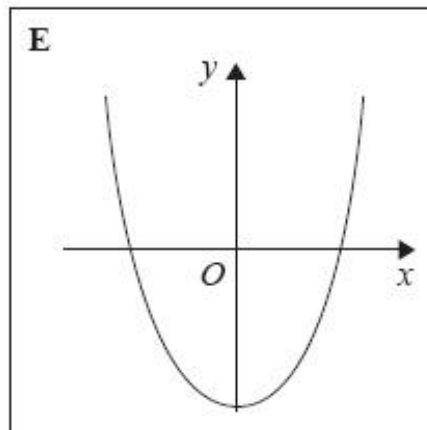
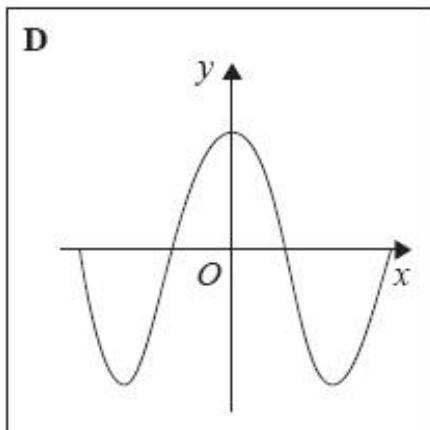
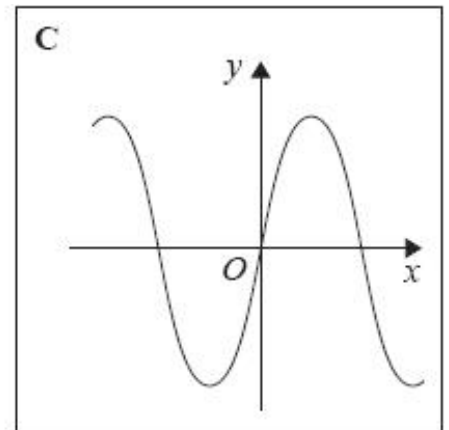
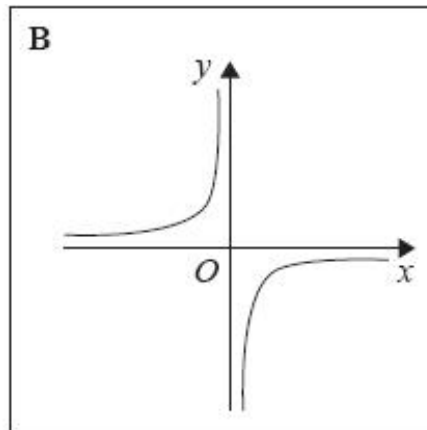
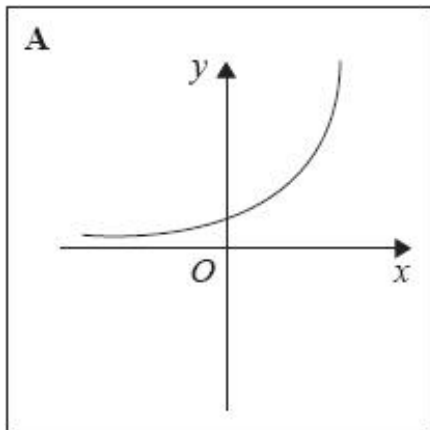
The mean mark for all the class is 60
The mean mark for the girls is 54

Work out the mean mark for the boys.

.....
(Total for question = 3 marks)

Q8.

Here are some graphs.



In the table opposite, match each equation with the letter of its graph.

Equation	Graph
$y = \sin x$	
$y = x^3 + 4x$	
$y = 2^x$	
$y = \frac{4}{x}$	

(Total for question = 3 marks)

Q9.

Using algebra, i.e. without a calculator, prove that $0.\dot{1}\dot{3}\dot{6} \times 0.\dot{2}$ is equal in value to $\frac{1}{33}$.

(Total for question = 3 marks)

Q10.

Richard invests £6000 for 5 years.
The investment gets compound interest of $x\%$ per annum.

At the end of 5 years the investment is worth £8029.35

Work out the value of x .

.....

(Total for question = 3 marks)

Q11.

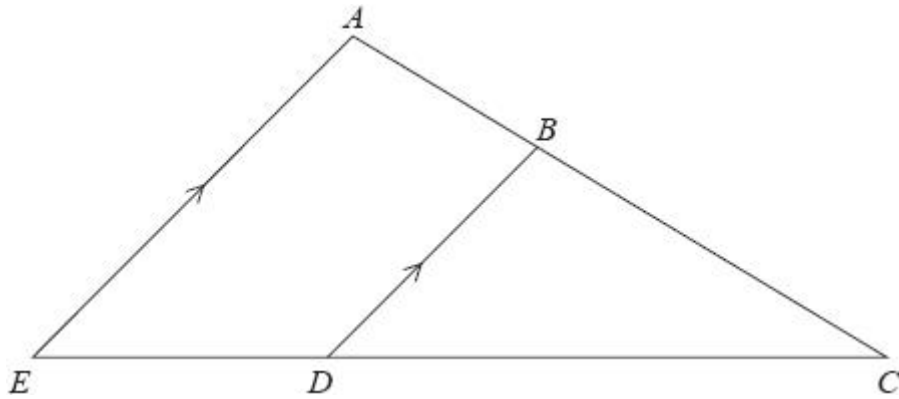
$$16^{1/5} \times 2^x = 8^{3/4}$$

Work out the exact value of x .

.....

(Total for question = 3 marks)

Q12.



ABC and EDC are straight lines.
 EA is parallel to DB .

$EC = 8.1$ cm.
 $DC = 5.4$ cm.
 $DB = 2.6$ cm.

(a) Work out the length of AE .

..... cm
(2)

$AC = 6.15$ cm.

(b) Work out the length of AB .

..... cm
(2)

(Total for question = 4 marks)

Q13.

The table shows some information about eight planets.

Planet	Distance from Earth (km)	Mass (kg)
Earth	0	5.97×10^{24}
Jupiter	6.29×10^8	1.898×10^{27}
Mars	7.83×10^7	6.42×10^{23}
Mercury	9.17×10^7	3.302×10^{23}
Neptune	4.35×10^9	1.024×10^{26}
Saturn	1.28×10^9	5.68×10^{26}
Uranus	2.72×10^9	8.683×10^{25}
Venus	4.14×10^7	4.869×10^{24}

(a) Write down the name of the planet with the greatest mass.

.....

(1)

(b) Find the difference between the mass of Venus and the mass of Mercury.

..... kg

(1)

Ben says that Neptune is over a hundred times further away from Earth than Venus is.

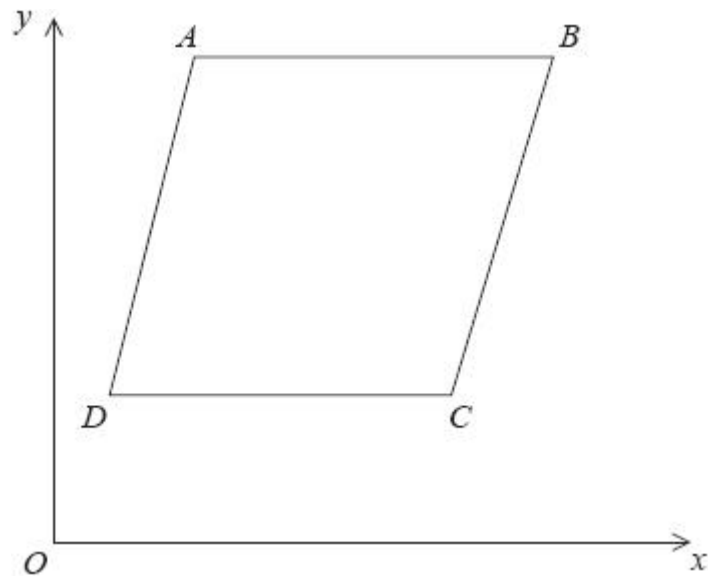
(c) Is Ben right?

You must show how you get your answer.

(2)

(Total for question = 4 marks)

Q14.



$ABCD$ is a rhombus.

The coordinates of A are $(5,11)$.

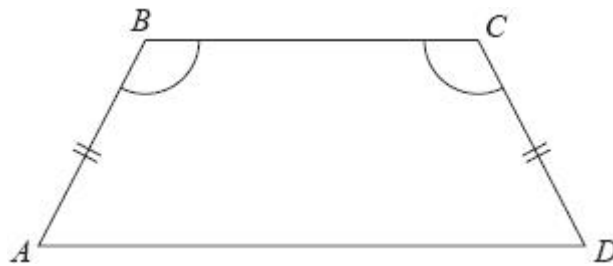
The equation of the diagonal DB is $y = \frac{1}{2}x + 6$

Find an equation of the diagonal AC .

.....
(Total for question = 4 marks)

Q15.

$ABCD$ is a quadrilateral.



$AB = CD$.

Angle $ABC =$ angle BCD .

Prove that $AC = BD$.

(Total for question = 4 marks)

Q16.

Solve

$$\frac{3x - 2}{4} - \frac{2x + 5}{3} = \frac{1 - x}{6}$$

$x =$

(Total for question = 4 marks)

Q17.

The table shows a set of values for x and y .

x	1	2	3	4
y	9	$2\frac{1}{4}$	1	$\frac{9}{16}$

y is **inversely** proportional to the square of x .

(a) Find an formula for y in terms of x .

.....
(2)

(b) Find the positive value of x when $y = 16$

.....
(2)

(Total for question = 4 marks)

Q18.

White shapes and black shapes are used in a game.

Some of the shapes are circles.

All the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is 3:7

The ratio of the number of white circles to the number of white squares is 4:5

The ratio of the number of black circles to the number of black squares is 2:5

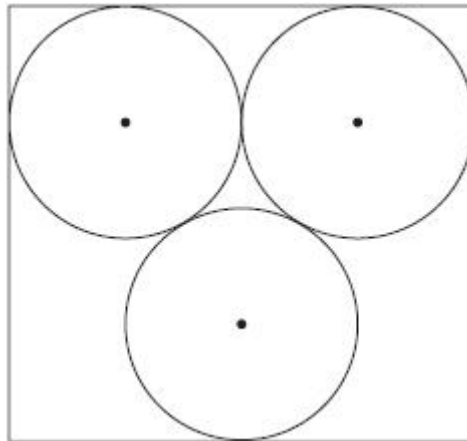
Work out what fraction of all the shapes are circles.

.....
(Total for question = 4 marks)

Q19.

The diagram shows 3 identical circles inside a rectangle.

Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram.



The radius of each circle is 24 mm.

Work out the area of the rectangle. Give your answer correct to 3 significant figures.

..... mm²

(Total for question = 4 marks)

Q20.

There are 9 counters in a bag.

7 of the counters are green.

2 of the counters are blue.

Emma takes at random two counters from the bag.

Work out the probability that Emma takes one counter of each colour.

You must show your working.

(Total for question = 4 marks)

Q21.

You are given that

$$2 - \frac{x+2}{x-3} - \frac{x-6}{x+3} \text{ can be written as a single fraction in the form } \frac{ax+b}{x^2-9}$$

where a and b are integers.

Work out the value of a and the value of b .

$a = \dots\dots\dots$

$b = \dots\dots\dots$

(Total for question = 4 marks)

Q22.

Marc bakes 420 cakes. He bakes only vanilla cakes, banana cakes, lemon cakes and chocolate cakes.

$\frac{2}{7}$ of the cakes are vanilla cakes.

35% of the cakes are banana cakes.

The ratio of the number of lemon cakes to the number of chocolate cakes is 4:5

Work out the number of lemon cakes Marc bakes.

$\dots\dots\dots$

(Total for question = 5 marks)

Q23.

Neil drove 56 km from Liverpool to Manchester.
He then drove 61 km from Manchester to Sheffield.

Neil 's average speed from Liverpool to Manchester was 70 km/h.
Neil took 75 minutes to drive from Manchester to Sheffield.

(a) Work out Neil's average speed for his total drive from Liverpool to Sheffield.

..... km/h
(4)

Jim drove from Barnsley to York.

Jim's average speed from Barnsley to Leeds was 80 km/h.
His average speed from Leeds to York was 60 km/h.

Jim says that the average speed from Barnsley to York can be found by working out the mean of 80 km/h and 60 km/h.

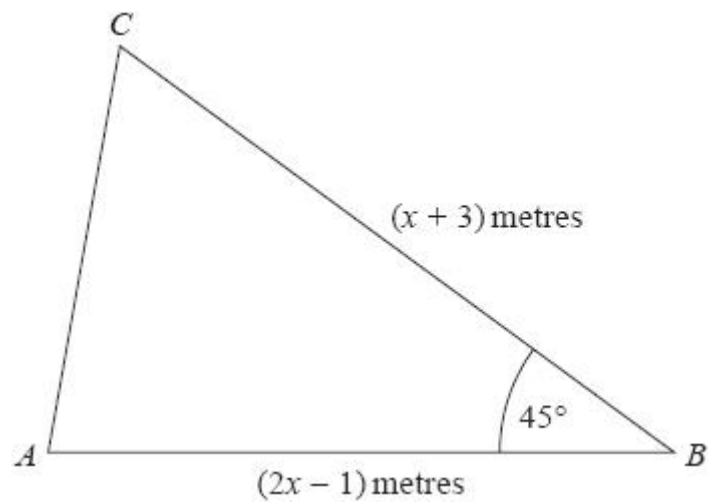
(b) If Jim is correct, what does this tell you about the two parts of Jim's journey?

.....
.....

(1)

(Total for question = 5 marks)

Q24.



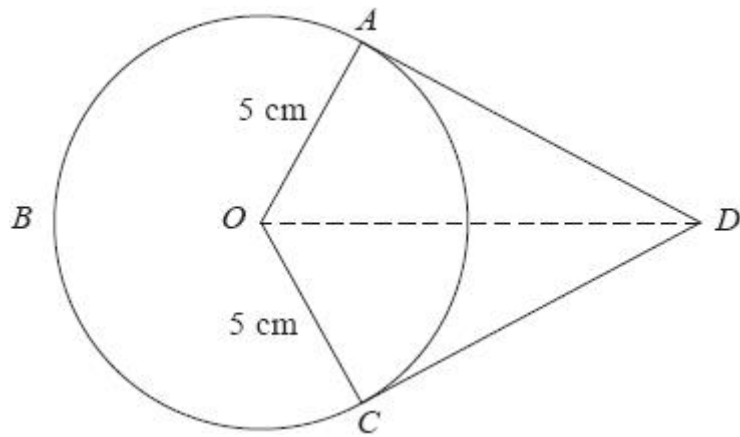
The area of triangle ABC is $6\sqrt{2} \text{ m}^2$.

Calculate the value of x .

Give your answer correct to 3 significant figures.

.....
(Total for question = 5 marks)

Q25.



A , B and C are points on a circle of radius 5 cm, centre O .

DA and DC are tangents to the circle.

$DO = 9$ cm

Work out the length of arc ABC .

Give your answer correct to 3 significant figures.

..... cm

(Total for question = 5 marks)

Q26.

(a) Simplify

$$\frac{x^2 - 16}{2x^2 - 5x - 12}$$

.....

(3)

(b) Make v the subject of the formula

$$w = \frac{15(t - 2v)}{v}$$

.....

(3)

(Total for question = 6 marks)

Q27.

Solve algebraically the simultaneous equations

$$x^2 + y^2 = 25$$

$$y - 3x = 13$$

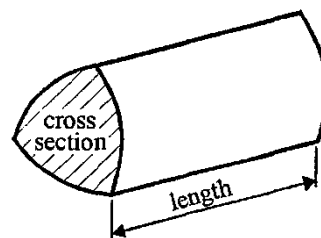
.....

(Total for question = 5 marks)

END OF PAPER. CHECK ALL YOUR WORK

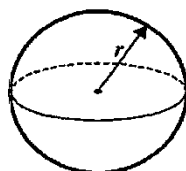
Formula Sheet

Volume of prism = area of cross-section \times length



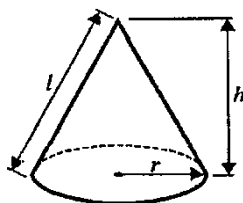
Volume of sphere = $\frac{4}{3} \pi r^3$

Surface area of sphere = $4\pi r^2$

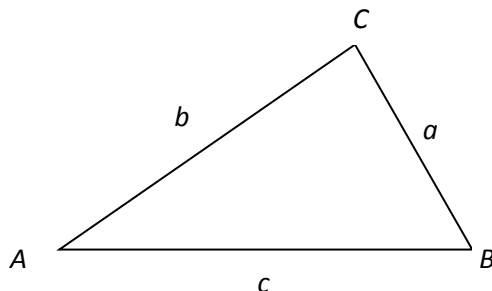


Volume of cone = $\frac{1}{3} \pi r^2 h$

Curved surface area of cone = $\pi r l$



In any triangle ABC



Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of a triangle = $\frac{1}{2} ab \sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$