Rugby School

Sixth Form Entrance Examination

Specimen Paper

MATHEMATICS 1

Time allowed: 60 minutes

Calculators are allowed, but not required.

Instructions to Candidates:

Write your solutions in the spaces provided.

Show all your workings clearly. Poorly set out work may be penalised.

Answer as many questions as you can. You may not be able to finish all the questions on the paper in the time available.

There are a total of 70 marks available.

Lined paper is available if needed.

1. (a) Simplify 4(2 – 3x)

_____ 1

(b) Expand and simplify $2x(x + 4) - (5 - 3x)$

_____ 2

(c) Factorise $4x^2 - 6x$

_____ 2

(d) Factorise $3x^2 - 10x - 8$

_____ 2

2.

ABCD is a rectangle. BC = 5cm and BD = 13cm.

(a) Find the length DC.

_____ 3

M is the point where the diagonals of the rectangle meet.

(b) Find the area of the triangle ABM.

2

3. A formula used in physics is $v = u + at$.

(a) Find v when $u = 5$, $a = -4$ and $t = 3$.

2

(b) If $v = 9u$ and $t = 2$, find a in terms of u.

2

In another formula $\frac{PV}{T} = nR$.

(c) Make T the subject of this formula.

3

4. A clothes shop decided to hold a sale, reducing prices by 20%.

(a) Work out the sale price of a shirt which had an original cost of £60;

2

(b) What was the original price of a jumper which costs £40 in the sale?

_____ 2

5. (a) Find the next two terms in the following sequences:

(i) 11, 8, 5, 2, ……..

_____ 2

(iii) -1, 0, 3, 8, ……..

_____ 2

(b) The nth term of a sequence is given by $\frac{2n+3}{2^n}$. Write down the 3rd and 4th terms of the sequence.

3rd term _____ 4th term _____ 2

(c) Find a formula for the nth term of the following sequences:

(i) 3, 5, 7, 9, ……..

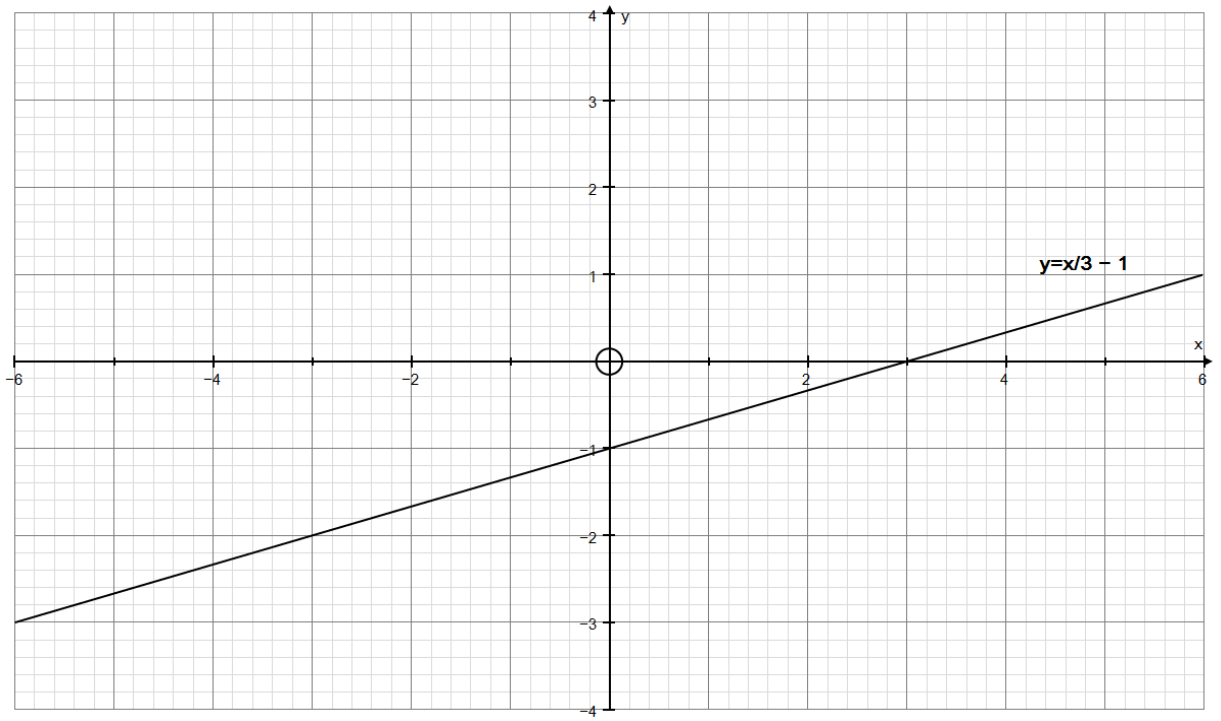
_____ 2

(ii) $\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}$, … … … … ..

_____ 2

6. What is the volume of a cylinder of diameter 10 cm, and height 20 cm (leave your answer in terms of π)? The formula for the volume of a cylinder is $V = \pi r^2 h$.

7.



(a) On the graph above the line with equation $y = \frac{x}{3} - 1$ has been drawn.

Add the lines $x + y = 3$ and $y = 2x + 2$. 4

(b) From your graph find the solution to the simultaneous equations $\begin{cases} y = \frac{x}{3} - 1 \\ x + y = 3 \end{cases}$. 2

(c) Use ***algebra*** to solve the simultaneous equations $\begin{cases} y = \frac{x}{3} - 1 \\ y = 2x + 2 \end{cases}$. Give your answers *as exact fractions*. 4

8. Solve for x :

(a) $3^{x+1} = \frac{1}{27}$

2

(b) $\frac{3}{x} = 24$

2

(c) $\frac{2}{x+1} + 4 = 9$

3

9. I decide to have a favourite photo, which has width 12 cm and length 15 cm, enlarged. The length of the enlarged photo is 80 cm.

(a) What is the width of the enlargement?

_____ 2

I also decide to buy a print of an original painting. The original painting is 45 cm high, and has an area of 2700 cm^2. The print is 30 cm high.

(b) What is the area of the print?

_____ 3

10. (a) A bag contains 9 balls numbered 1, 2, 3, …., 9. The balls which have an even number are coloured blue; the remainder are white. I take a ball from the bag, replace it and then take a second ball. What is the probability

(i) that both balls are blue?

_____ 2

(ii) that the total score on the two balls is 17?

_____ 3

(b) Two faces of a cube are selected at random. What is the probability that they are opposite faces of the cube?

_____ 2

11. ***Only answer these questions if you have finished all the previous questions. This work may seem unfamiliar.***

For this question we let a ◊ b = $a^2 + b^2$. So, for example, 2 ◊ 3 = $2^2 + 3^2 = 13$.

(a) Work out 3 ◊ (-3).

_____ 1

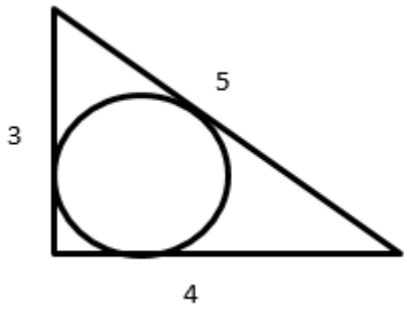
(b) Work out 2 ◊ (3 ◊ 4).

_____ 1

(c) Solve the equation $x \lozenge x = 3x + 9$.

_____ 1

12. The diagram below shows a right-angled triangle whose lengths are 3 , 4 and 5 units. A circle of radius r is drawn inside the triangle so that its circumference just touches all three sides:



*(Diagram **not** drawn accurately)*

Showing all of your working, find the exact area of the circle.
You should leave your answer in terms of π.

_____ 2

END OF EXAMINATION

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16+ Scholarship — Specimen Paper — Mathematics 1

1. (i) $8-12x$

   (ii) $2x^2+8x-5+3x = 2x^2+11x-5$

   (iii) $2x(2x-3)$

   (iv) $(3x+2)(x-4)$

2. (i) Using Pythagoras' Theorem, $DC = \sqrt{13^2-5^2} = 12$cm.

   (ii) Since $BC = 5$ cm, and M is half-way from A to C (and from B to D), the vertical height of M above the base AB (of the triangle ABM is 2.5 cm. Thus the area of ABM is $\frac{1}{2} \times 2.5 \times 12 = 15$ cm^2

3. (i) $v = 5+(-4)\times 3 = 5-12 = 7$

   (ii) $9u = u+2a$, and hence $8u = 2a$, so that $a = 4u$.

   (iii) We manipulate to obtain

$$\begin{aligned} \frac{PV}{T} &= nR \\ PV &= nRT \\ \frac{PV}{nR} &= T \end{aligned}$$

4. (i) The sale price is £60×0.8, or £48.

   (ii) If the original price is £x, then $0.8x = 40$, and hence $x = 50$. The original price was £50.

5. (i) (a) $11, 8, 5, 2, -\mathbf{1}, -\mathbf{4}, \ldots$ — numbers decrease by 3 each time.

   (b) $-1, 0, 3, 8, \mathbf{15}, \mathbf{35}, \ldots$ — successive squares minus 1.

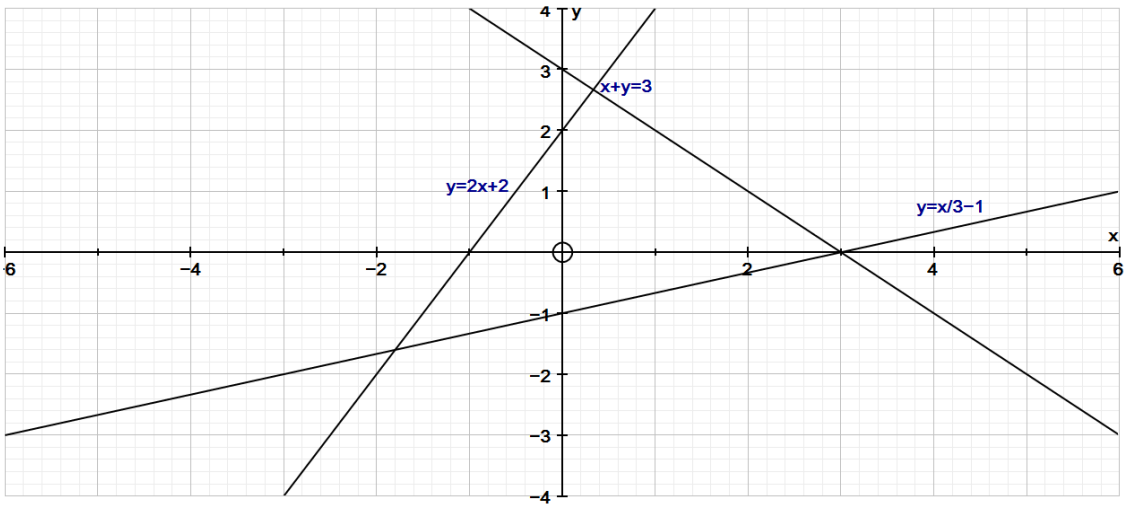
   (ii) $\frac{9}{8}$ and $\frac{11}{16}$.

   (iii) (a) $2n+1$,

   (b) $\frac{n^2}{n+1}$

6. The volume is $\pi \times 5^2 \times 20 = 500\pi$ cm^3.

7. (i)



(ii) The lines $y = \frac{1}{3}x - 1$ and $x + y = 3$ meet at the point $(3, 0)$.

(iii) Substituting, we obtain

$$\begin{aligned} \tfrac{1}{3}x - 1 &= 2x + 2 \\ -1 - 2 &= 2x - \tfrac{1}{3}x \\ \tfrac{5}{3}x &= -3 \end{aligned}$$

and hence $x = -\frac{9}{5}$, and hence $y = -\frac{8}{5}$.

8. (i) We have $3^{x+1} = \frac{1}{27} = 3^{-3}$, so $x + 1 = -3$, so $x = -4$.

(ii) We have $3 = 24x$, so $x = \frac{1}{8}$.

(iii) Solving,

$$\begin{aligned} \frac{2}{x+1} + 4 &= 9 \\ \frac{2}{x+1} &= 5 \\ 2 &= 5(x+1) \\ -3 &= 5x \end{aligned}$$

and hence $x = -\frac{3}{5}$.

9. (i) The enlargement has scale factor $\frac{80}{15} = \frac{16}{3}$. Thus the enlarged width is $12 \times \frac{16}{3} = 64$ cm.

(ii) This enlargement has scale factor $\frac{30}{45} = \frac{2}{3}$. Thus the area of the print is $2700 \times \left(\frac{2}{3}\right)^2 = 1200$ cm^2.

10. (i) (a) $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$.

(b) To score 17, I must choose 8 and 9 in some order. The probability of doing this is $\frac{1}{9} \times \frac{1}{9} + \frac{1}{9} \times \frac{1}{9} = \frac{2}{81}$.

(ii) Once the first face has been chosen, there are five faces, out of which we will choose the second face. Only one of these faces is opposite the first face. Thus the probability that the two faces are opposite each other is $\frac{1}{5}$.

11. (i) $3 \diamond (-3) = 3^2 + (-3)^2 = 18$.

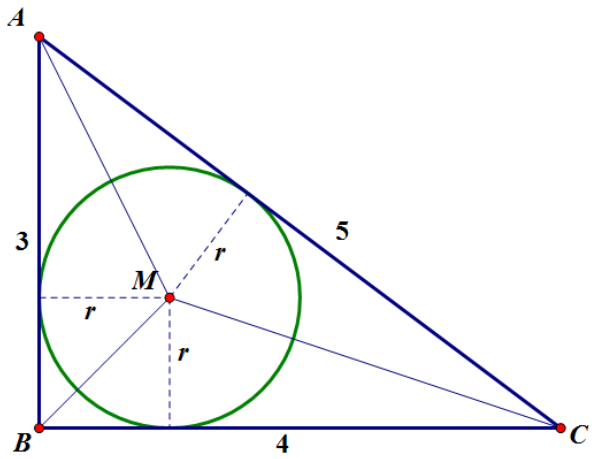
(ii) $2 \diamond (3 \diamond 4) = 2 \diamond (3^2 + 4^2) = 2 \diamond 25 = 2^2 + 25^2 = 629$.

(iii)

$$\begin{aligned} x \diamond x &= 3x + 9 \\ x^2 + x^2 &= 3x + 9 \\ 2x^2 - 3x - 9 &= 0 \\ (2x + 3)(x - 3) &= 0 \end{aligned}$$

and hence $x = -\frac{3}{2}, 3$.

12. Let M be the centre of the inscribed circle. The triangles ABM, BCM, CAM have bases $AB = 3$, $BC = 4$ and $CA = 5$, and heights r above their bases.



Thus these triangles have areas $\frac{3}{2}r$, $2r$ and $\frac{5}{2}r$ respectively. The sum of their areas is $6r$, which must equal the total area of the triangle ABC, which is $\frac{1}{2} \times 3 \times 4 = 6$. Thus $r = 1$, and so the area of the inscribed circle is $\pi r^2 = \pi$.