



Rugby School

Sixth Form Entrance Examination

Specimen Paper

MATHEMATICS 2

Time allowed: 60 minutes

Calculators are allowed.

Instructions to Candidates:

Write your solutions in the spaces provided.

Show all your workings clearly. Poorly set out work may be penalised.

Answer as many questions as you can. You may not be able to finish all the questions on the paper in the time available.

There are a total of 70 marks available.

Lined paper is available if needed.

1.



ABCD is a rectangle. $BC = 4\text{cm}$ and $BD = 20\text{cm}$.

Give all answers in this question to 3 significant figures.

a) Find the length DC.

2

M is the point where the diagonals of the rectangle meet.

b) Find the area of the triangle ABM.

2

2. A formula used in physics is $v = u + at$.

a) If $v = 9u$ and $t = 2$, find a in terms of u .

2

b) Make x the subject of:

$$y = \frac{ax+b}{cx+d}$$

2

3. a) Find the next two terms in the sequence $1, \frac{3}{2}, \frac{9}{4}, \frac{27}{8}, \dots \dots \dots$

_____ 2

b) The n th term of a sequence is given by $\frac{2n+3}{2^n}$. Write down the 3rd and 4th terms of the sequence.

3rd term _____ 4th term _____ 2

c) Find a formula for the n th term of the sequence $\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \dots \dots \dots$

_____ 2

4. Alice and Ben have to shell the peas for lunch. If Alice worked on her own, as quickly as possible, she would take two hours to shell the peas. If Ben worked on his own, as quickly as possible, he would take three hours to shell the peas. In fact, both share the work.

a) How quickly can they shell the peas?

b) What fraction of the peas will be shelled by Alice? _____ 4

_____ 2

5. I invest £400 in a savings account at my bank. The account pays 2.25% interest over the year.

Then, because I have been such a good customer I am then offered 2.75% interest per year as long as long as I leave the money untouched for two more years. The interest is compound interest.

a) How much will I have in the bank after the three years?

3

My friend thinks she can do better and she chooses to put her £400 into a different account which pays 0.2% compound interest each month.

b) How much will she have in the bank after three years? Give your answer to the nearest penny.

3

6. A bag contains 9 balls numbered 1, 2, 3, ..., 9. The balls which have an even number are coloured blue; the remainder are white. I take a ball from the bag, replace it and then take a second ball. What is the probability

a) that both balls are blue?

_____ 2

b) that the total score on the two balls is 17?

_____ 3

c) both balls are white and the total score on the two balls is even?

_____ 2

7. For this question we let $a \diamond b = a^2 + b^2$. So, for example, $2 \diamond 3 = 2^2 + 3^2 = 13$.

a) Work out $3 \diamond (-3)$.

_____ 2

b) Work out $2 \diamond (3 \diamond 4)$.

_____ 2

c) Solve the equation $x \diamond x = 3x + 9$.

_____ 2

8. Write $x^2 + 8x - 4$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

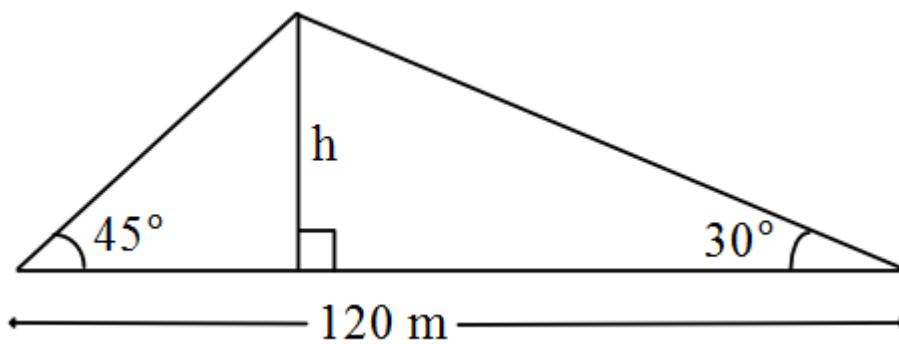
2

Hence find the minimum value of this quadratic expression and the value of x for which this occurs.

2

9.

Find the value of h , giving your answer to 3 significant figures:
(the diagram is not to scale)



4

10. a) Simplify $\frac{x^2-y^2}{(x-y)^2}$

2

b) Express as a single fraction: $\frac{2x-4}{15} + \frac{x}{5} - \frac{x-1}{3}$

3

11. Solve the three simultaneous equations below

$$2x - y + z = -1$$

$$x + 2y - z = 6$$

$$3x + 5y - 7z = 20$$

5

12. A school expedition is rather badly organised. There are N children signed up for the trip. One fifth of them are left behind at school. Eight are misled at a service station on the way. Half the remainder are lost on the expedition. Five more are misled at a service station on the way back.

a) Write an expression in N for the number of children who make it back to school.

_____ 3

In fact, one quarter of the children who signed up for the trip make it back to school.

b) Find N .

_____ 3

13. a) By substituting $x = 1 + \frac{1}{p}$ into the equation $2x^4 + x^3 - 6x^2 + x + 2 = 0$, show that $9p^2 + 9p + 2 = 0$.

3

- b) Hence solve the equation $2x^4 + x^3 - 6x^2 + x + 2 = 0$ fully.

4

End of Examination

16+ Scholarship — Specimen Paper — Mathematics 2

1. (i) By Pythagoras' Theorem, $DC = \sqrt{20^2 - 4^2} = \sqrt{384} = 19.6$ cm.
(ii) Since $BC = 4$ cm, the vertical height of M "above" the base AB is 2 cm, so the area of ABM is $\frac{1}{2} \times AB \times 2 = 19.6$ cm².
2. (i) $9u = u + 2a$, and so $2a = 8u$, so $a = 4u$.
(ii) Manipulating,

$$\begin{aligned}y &= \frac{ax+b}{cx+d} \\(cx+d)y &= ax+b \\cxy+dy &= ax+b \\cxy-ax &= b-dy \\x(cy-a) &= b-dy \\x &= \frac{b-dy}{cy-a}\end{aligned}$$

3. (i) $\frac{81}{16}$ and $\frac{243}{32}$ (powers of $\frac{3}{2}$).
(ii) $\frac{9}{8}$ and $\frac{11}{16}$.
(iii) $\frac{n^2}{n+1}$.
4. (i) Alice shells $\frac{1}{2}$ the peas in an hour. Ben shells $\frac{1}{3}$ of the peas in an hour. Together they shell $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ of the peas in an hour, and so they shell all the peas in $\frac{6}{5}$ hours, or 1 hour and 12 minutes.
(ii) In that time, Alice shells $\frac{6}{5} \times \frac{1}{2} = \frac{3}{5}$ of the peas. Alternatively, since the times Alice and Ben take to shell the same amount of peas is in the ratio 2 : 3, the number of peas they shell in a fixed time is in the ratio 3 : 2, and so Alice will always shell $\frac{3}{5}$ of the peas that the two of them shell in any time period.
5. (i) After 3 years I will have $\pounds 400 \times 1.0225 \times 1.0275^2 = \pounds 431.80$ to the nearest penny.
(ii) After 36 months the friend will have $\pounds 400 \times 1.002^{36} = \pounds 429.83$ to the nearest penny.
6. (i) $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$.
(ii) To score 17, I must choose 8 and 9 in some order. The probability of doing this is $\frac{1}{9} \times \frac{1}{9} + \frac{1}{9} \times \frac{1}{9} = \frac{2}{81}$.
(iii) Since all white balls have odd numbers on them, if both balls chosen are white, then the sum of the numbers of them must be even! Thus we are simply interested in the probability that both balls are white, which is $\frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$.
7. (i) $3 \diamond (-3) = 3^2 + (-3)^2 = 18$.
(ii) $2 \diamond (3 \diamond 4) = 2 \diamond (3^2 + 4^2) = 2 \diamond 25 = 2^2 + 25^2 = 629$.

(iii)

$$\begin{aligned}x \diamond x &= 3x + 9 \\x^2 + x^2 &= 3x + 9 \\2x^2 - 3x - 9 &= 0 \\(2x + 3)(x - 3) &= 0\end{aligned}$$

and hence $x = -\frac{3}{2}, 3$.

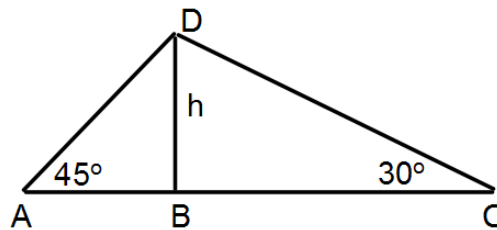
8. We can write

$$x^2 + 8x - 4 = (x + 4)^2 - 16 - 4 = (x + 4)^2 - 20$$

Since $(x + 4)^2$ is a perfect square, $(x + 4)^2 \geq 0$ for all real x , and so $x^2 + 8x - 4 \geq -20$. The minimum possible value is -20 , and this occurs when $x + 4 = 0$, so when $x = -4$.

9. The right-angled triangle ABD is isosceles, and hence $AB = h$. Considering the right-angled triangle BCD , we deduce that

$$\frac{h}{BC} = \tan 30^\circ$$



Thus

$$\begin{aligned}AB + BC &= 120 \\h + \frac{h}{\tan 30^\circ} &= 120 \\h \tan 30^\circ + h &= 120 \tan 30^\circ \\h(\tan 30^\circ + 1) &= 120 \tan 30^\circ \\h &= \frac{120 \tan 30^\circ}{\tan 30^\circ + 1}\end{aligned}$$

and so $h = 43.9$ m (to 3 SF).

10. (i) We have

$$\frac{x^2 - y^2}{(x - y)^2} = \frac{(x - y)(x + y)}{(x - y)^2} = \frac{x + y}{x - y}$$

(ii) Putting everything over the common denominator 15,

$$\frac{2x - 4}{15} + \frac{x}{5} - \frac{x - 1}{3} = \frac{(2x - 4) + 3x - 5(x - 1)}{15} = \frac{1}{15}$$

11. Eliminating x from two of the equations, and then eliminating y from one of these, shows that

$$\begin{array}{rcl} x + 2y - z = 6 & & x + 2y - z = 6 & & x + 2y - z = 6 \\ 2x - y + z = -1 & \Leftrightarrow & -5y + 3z = -13 & \Leftrightarrow & y + 4z = -2 \\ 3x + 5y - 7z = 20 & & -y - 4z = 2 & & 23z = -23 \end{array}$$

from which we deduce that $z = -1$, $y = 2$ and $x = 1$. The solution is $(1, 2, -1)$.

12. (i) $\frac{4}{5}N$ start out from school, $\frac{4}{5}N - 8$ are left after the service station, $\frac{1}{2}(\frac{4}{5}N - 8) = \frac{2}{5}N - 4$ do not get lost, and so a total of $\frac{2}{5}N - 4 - 5 = \frac{2}{5}N - 9$ make it back to school.

(ii) We need to solve the equation

$$\begin{aligned} \frac{2}{5}N - 9 &= \frac{1}{4}N \\ \frac{2}{5}N - \frac{1}{4}N &= 9 \\ \frac{3}{20}N &= 9 \end{aligned}$$

and so $N = 60$.

13. (i) We have

$$\begin{aligned} x &= 1 + p^{-1} \\ x^2 &= 1 + 2p^{-1} + p^{-2} \\ x^3 &= 1 + 3p^{-1} + 3p^{-2} + p^{-3} \\ x^4 &= 1 + 4p^{-1} + 6p^{-2} + 4p^{-3} + p^{-4} \end{aligned}$$

and hence the equation becomes

$$\begin{aligned} 2x^4 + x^3 - 6x^2 + x + 2 &= 0 \\ 2 + 8p^{-1} + 12p^{-2} + 8p^{-3} + 2p^{-4} + 1 + 3p^{-1} + 3p^{-2} + p^{-3} &= 0 \\ -6 - 12p^{-1} - 6p^{-2} + 1 + p^{-1} + 2 & \\ 9p^{-2} + 9p^{-3} + 2p^{-4} &= 0 \\ 9p^2 + 9p + 2 &= 0 \end{aligned}$$

(ii) We solve

$$\begin{aligned} 9p^2 + 9p + 2 &= 0 \\ (3p + 1)(3p + 2) &= 0 \end{aligned}$$

and hence $p = -\frac{1}{3}, -\frac{2}{3}$. Substituting these values into the formula $x = 1 + p^{-1}$ gives the solutions $x = -2, -\frac{1}{2}$. Thus $x + 2$ and $2x + 1$ are factors of the quartic polynomial, and we calculate

$$2x^4 + x^3 - 6x^2 + x + 2 = (2x + 1)(x^3 - 3x + 2) = (2x + 1)(x + 2)(x^2 - 2x + 1) = (2x + 1)(x + 2)(x - 1)^2$$

Thus the complete solution of $2x^4 + x^3 - 6x^2 + x + 2 = 0$ is $x = -\frac{1}{2}, -2, -1$ (twice).