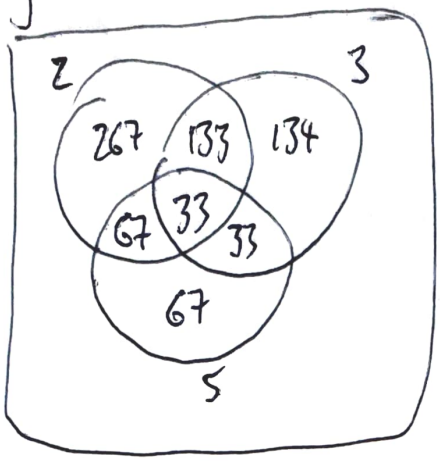
Sevenoaks 2021 HL

1) div by 2 : 500
div by 3 : 333
div by 5 : 200

div by $2\times3=6$: 166
div by $2\times5=10$: 100
div by $3\times5=15$: 66

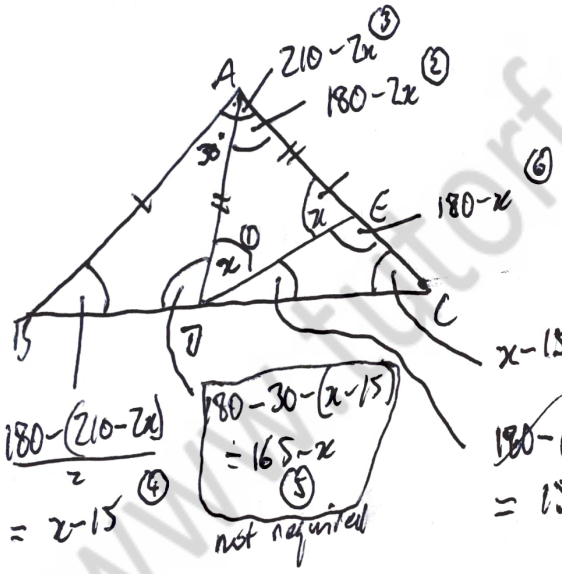
div by $2\times3\times5=30$: 33



Not div by any:
$1000 - 500 - 134 - 33 - 67$
$= 266$

Or use inclusion-exclusion principle
$1000 - (500 + 333 + 200 - 166 - 100 - 66 + 33)$
$= 266.$

2)



3)
$x^2 - 3x + 1 = 0$

$x = \frac{3 \pm \sqrt{9-4}}{2}$

$= \frac{3 \pm \sqrt{5}}{2}$

$x^2 = \frac{1}{4}(9 + 5 \pm 6\sqrt{5})$

$= \frac{1}{4}(14 \pm 6\sqrt{5})$

$= \frac{1}{2}(7 \pm 3\sqrt{5})$

$\frac{1}{x^2} = \frac{2}{7 \pm 3\sqrt{5}}$

$= \frac{2(7 \mp 3\sqrt{5})}{(7 \pm 3\sqrt{5})(7 \mp 3\sqrt{5})}$

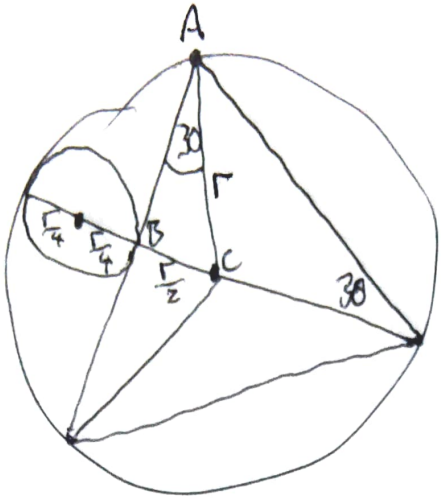
$= \frac{14 \mp 6\sqrt{5}}{49 - 45}$

$= \frac{14 \mp 6\sqrt{5}}{4}$

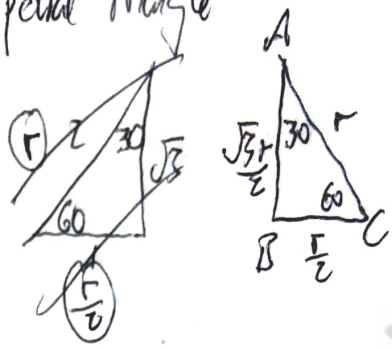
$= \frac{1}{2}(7 \mp 3\sqrt{5})$

So $x^2 + \frac{1}{x^2}$
$= \frac{1}{2}(7 \pm 3\sqrt{5}) + \frac{1}{2}(7 \mp 3\sqrt{5})$
$= 7$

4)



Special triangle



$$\frac{\text{Area large}}{\text{Area small}} = \frac{\pi r^2}{\pi\left(\frac{r}{4}\right)^2}$$

$$= 16$$

5) 3 consecutive odd numbers squared and added:

$$(2n-1)^2 + (2n+1)^2 + (2n+3)^2$$

$$4n^2 - 4n + 1 + 4n^2 + 4n + 1 + 4n^2 + 12n + 9$$

$$= 12n^2 + 12n + 11$$

$$= 12(n^2 + n) + 11$$

which is a multiple of 12 plus 11, as required.