

Sevensonky 2022 HL

1)

$$\frac{u_5}{u_{12}} = \frac{6}{13}$$

$$\frac{u_1 + 4d}{u_1 + 11d} = \frac{6}{13}$$

$$13u_1 + 52d = 6u_1 + 66d$$

$$7u_1 = 14d$$

$$u_1 = 2d$$

$$u_1 \times u_3 = 32$$

$$u_1 \times (u_1 + 2d) = 32$$

$$u_1 \times 2u_1 = 32$$

$$u_1 = 4$$

$$d = 2$$

$$S = \frac{1}{2}n(2a + (n-1)d)$$

$$= 50(4 + 99 \times 2)$$

$$= 50 \times 202$$

$$= 10100$$

$$2) \quad x^4 - y^4 = 2009 \quad x^2 + y^2 = 49$$

$$(x^2 - y^2)(x^2 + y^2) = 2009$$

$$49(x^2 - y^2) = 2009$$

$$x^2 - y^2 = 41$$

$$x^2 + y^2 = 49$$

$$\hline 2x^2 = 90$$

$$x^2 = 45$$

$$x = 3\sqrt{5}$$

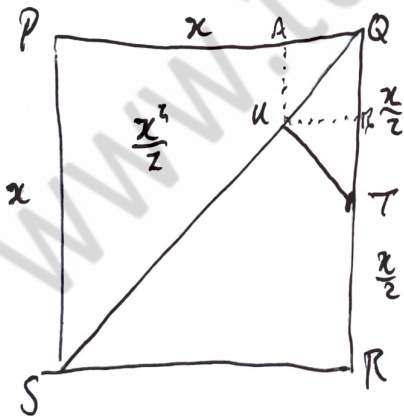
$$y^2 = 49 - 45$$

$$y^2 = 4$$

$$y = 2$$

$$\frac{y}{x} = \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{15} = \frac{2}{15}\sqrt{5}$$

3)



$$UT = UQ = \frac{x}{2\sqrt{2}}$$

$$\begin{aligned} \text{Area } QUT &= \frac{1}{2} \times \left(\frac{x}{2\sqrt{2}}\right)^2 \\ &= \frac{1}{2} \times \frac{x^2}{8} \\ &= \frac{x^2}{16} \end{aligned}$$

(or say Area $QUT = \text{area } AUQ = \frac{1}{16}x^2$)

$$SU\overline{TR} = x^2 - \frac{x^2}{2} - \frac{x^2}{16}$$

$$= \frac{7x^2}{16}$$

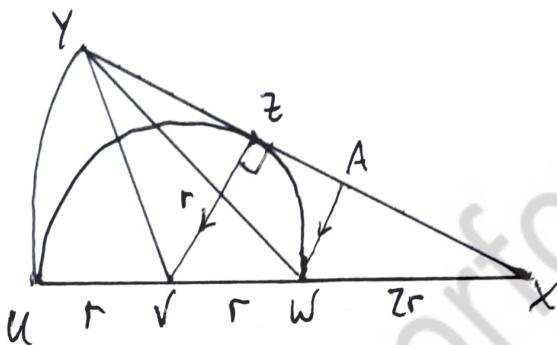
4) 7, 13, 19, 25

$$u_k = 6k + 1 \quad u_k^2 = (6k+1)^2 = 36k^2 + 12k + 1$$

$$u_{k+1} = 6(k+1) + 1 \quad u_{k+1}^2 = (6(k+1)+1)^2 = (6k+7)^2 = 36k^2 + 84k + 49$$

$$u_{k+1}^2 - u_k^2 = 72k + 48 = 24(3k+2) \text{ so a multiple of } 24$$

5)



Put A so that AW is parallel to ZV.

$$AW = \frac{2}{3} ZV = \frac{2}{3} r$$

$$\begin{aligned} \text{Area } XYZ &= \frac{1}{2} \text{ base} \times \text{height} \\ &= \frac{1}{2} \times XY \times ZV \\ &= \frac{1}{2} \times 4r \times r \\ &= 2r^2 \end{aligned}$$

$$\begin{aligned} \text{Area } XYW &= \frac{1}{2} \text{ base} \times \text{height} \\ &= \frac{1}{2} \times XY \times AW \\ &= \frac{1}{2} \times 4r \times \frac{2}{3}r \\ &= \frac{4}{3}r^2 \end{aligned}$$

$$\begin{aligned} \text{Area } YVW &= XYZ - XYW \\ &= 2r^2 - \frac{4}{3}r^2 \\ &= \frac{2}{3}r^2 \end{aligned}$$