First name:	Surname:

Current School:
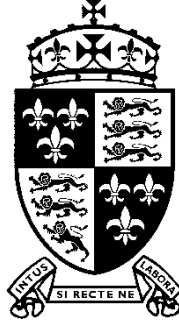
SHREWSBURY SCHOOL

SIXTH FORM ENTRANCE EXAMINATION
FOR ENTRY IN SEPTEMBER 2022

MATHEMATICS

(1 Hour 15 Minutes)

Instructions to candidates:

Answer all questions, writing your answers in the spaces provided.

The number of marks for each question is shown in square brackets: [ ].

Section A contains questions of a GCSE nature. Attempt this section first, but do not spend too long on any particular question.

Section B is intended to be considerably more difficult, and is mainly targeted at candidates who are aiming for an academic scholarship on the strength of their mathematics, or who are hoping to take Further Mathematics at A-Level.

You are expected to use a calculator in this examination.

Relevant working must be shown in order to gain high marks.
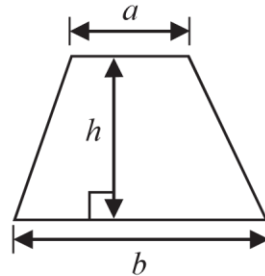
Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\,[2a + (n-1)d]$
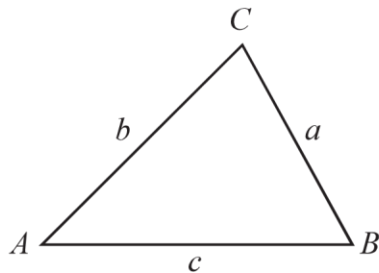
The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \dfrac{1}{2}(a+b)h$
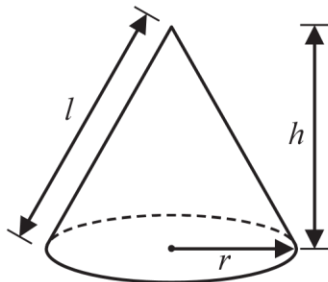

Trigonometry

In any triangle ABC

Sine Rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $\quad a^2 = b^2 + c^2 - 2bc \cos A$

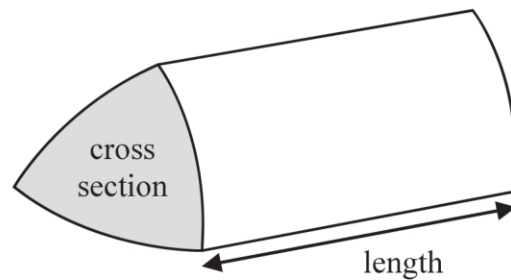**Area of triangle** $= \dfrac{1}{2}ab \sin C$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

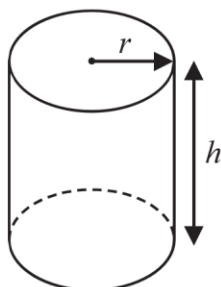**Curved surface area of cone** $= \pi r l$

Volume of prism
= area of cross section × length

Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi r h$

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
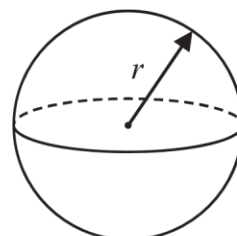
Section A (60 marks)

Answer all questions in this section.

1) Expand and simplify the following expressions:

   a) $4(2a + 3b) - 2(7a - 5b)$

   _____ [2]

   b) $(3x + 5)(x - 2)$

   _____ [2]

2) Factorise the following expressions fully:

   a) $4cd^2 + 6c^2d$

   _____ [2]

   b) $t^2 - 8t + 15$

   _____ [2]

3) You must **not** use a calculator in this question. Full working **must** be shown.

a) Evaluate these, giving your answers as **mixed numbers** in their simplest form:

i) $4\frac{2}{3} + 2\frac{1}{4}$

_____ [2]

ii) $4\frac{2}{3} \times 2\frac{1}{4}$

_____ [3]

b) Simplify the following:

$$(2t^3 \times 4t^7)^2$$

_____ [3]

4) a) In a difficult Norwegian exam, Morten scores only 33 marks out of 120. What is this as a percentage?

_____ [2]

    b) A record shop is having a sale, with all prices of classical CDs reduced by 45%. A splendid CD of Bruckner's 8th symphony is on sale for only £6.49.

    What is the normal selling price of this CD?

_____ [2]

    c) One Bitcoin was worth $29,000 at the start of the year, and rose to a value of $67,000 in October.

    What was the percentage increase in the value of one Bitcoin during this period? Give your answer to 3 significant figures.

_____ [2]

5) The diagram below shows a door wedge in the form of a prism of depth 6 cm, whose cross-section is a right-angled triangle with base 10 cm and perpendicular height 7.5 cm.
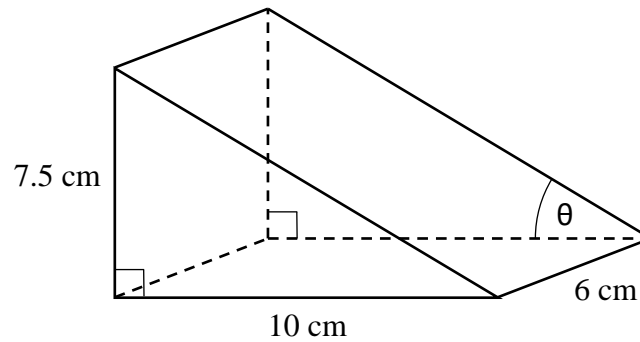
a) Calculate the volume of the door wedge, giving the correct units in your answer.

_____ [3]

b) Calculate the total surface area of the door wedge, giving the correct units in your answer.

_____ [5]

c) Calculate the angle θ between the sloping face and the base of the door wedge to 3 significant figures.

_____ [2]

6) The drag force D on a toy rocket is directly proportional to the **square** of its velocity. When $v = 4$ ms^{-1}, $D = 20$ Newtons.

a) Find an exact formula for D in terms of v.

_____ [3]

b) Calculate the velocity of the rocket when the drag force is 50 Newtons. Give your answer to 3 significant figures.

_____ [3]

7) Rearrange the following formulae to make *x* the subject:

a) $4x + 3y = 5$

_____ [2]

b) $P = \dfrac{2}{x+5}$

_____ [3]

c) $g = 3(x - 4)^2$

_____ [3]

d) $H = \dfrac{x+2}{x-4}$

_____ [3]

8) Solve the following equations:

   a) $7x - 4 = 2x + 11$

_____ [2]

   b) $\dfrac{2x+5}{8} = \dfrac{x}{3}$

_____ [3]

   c) $3x^2 - 16x + 5 = 0$

_____ [3]

d) $(x^2 - 3)^{2021} = 4(x^2 - 3)^{2019}$

_____ [3]

Spare page for Section A working.

Section B (20 marks)

This section is intended to be considerably more difficult, and is targeted at candidates who are aiming for an academic scholarship on the strength of their mathematics, or who are hoping to take Further Mathematics at A-Level.

Only attempt these questions if you have done as much of Section A as you can.

B1) a) Expand and simplify $(x - 2)^2$.

_____ [1]

b) List all the pairs of integer (whole number) values of x and y which are solutions to the equation
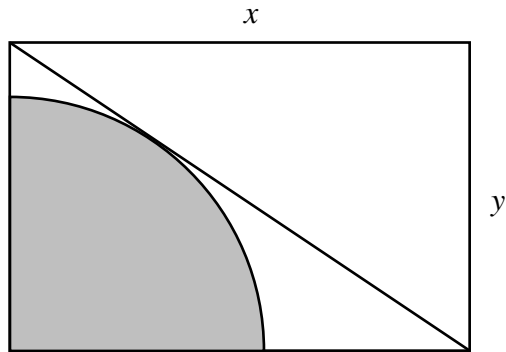
$$x^2 + y^2 = 1$$

_____ [2]

c) Find all the pairs of integer values of x and y which are solutions to the equation

$$x^2 + y^2 = 4x - 8y - 16$$

_____ [4]

B2)   The diagram below shows a quarter-circle inside a rectangle of width x and height y. The radii of the quarter-circle lie along adjacent edges of the rectangle, and the arc touches a diagonal of the rectangle.

Find and simplify an expression in terms of x and y for the proportion of the rectangle which is occupied by the quarter-circle.

(Hint: you may find it helpful to consider similar triangles.)

_____ [6]

Please turn over for the final question.

B3) Eric Cartman is preparing his 3 lunchboxes for a day out on a school trip. The first lunchbox is red, the second is green, and the third is blue.

He has the following items to pack into his lunchboxes:

- 3 different sandwiches (Avocado, Bacon, Chicken);
- 3 different chocolate bars (KitKat, Lion, Mars);
- 3 different drinks (Nesquik, Orangina, Pepsi).

a) How many different ways are there for Cartman to pack the lunchboxes if each box must contain exactly one sandwich, one drink and one chocolate bar?

(For example, one way is to pack A, K, N into the red box, B, L, O into the green box, and C, M, P into the blue box.)

_____ [3]

b)   How many different ways are there for him to pack the lunchboxes if each box can contain **any** three items?

_____ [4]

+++++ END +++++

Spare page for Section B working.