St Swithun's 2022 16+ entrance

1) $45°$

2) $x \geq \frac{15}{7} = 2\frac{1}{7}$

3) $6 - (3 - 7abx)^2$

4) $\left(\sqrt{27} - \sqrt{3}\right)^2 = \left(3\sqrt{3} - \sqrt{3}\right)^2 = \left(\sqrt{3}\right)^2(3-1)^2 = 3(2)^2 = 12$

5) a) $x = \frac{1}{7}$

b) $x = \frac{50}{34} = 1\frac{16}{34} = 1\frac{8}{17}$

c) $x = -6 \; or \; 8$

6) $\frac{\frac{11}{2} \times \frac{38}{33}}{\frac{19}{4}} = \frac{11 \times 38 \times 4}{2 \times 33 \times 19} = \frac{2 \times 2}{3} = 1\frac{1}{3}$

7) $y = \frac{3}{11}x - \frac{2}{11}$

8) 31

9) $\frac{2(2n^2+3n-1)}{(2n-1)(2n+3)}$

10) $\pm 23, \; \pm 10, \; \pm 5, \; \pm 2$

11) It's a quadratic, symmetrical for a vertical line through the x-coordinate required. The x-coordinate required is halfway between the roots, which are 3 and -5. Halfway between those values is x=-1.
Or: expand and complete the square.

12) $7 + 2\sqrt{5}$

13) $a = 1,2,3$ so as not to exceed 19. a=1 gives b=14/3. a=2 gives b=3. a=3 gives b=4/3. So the only solution is a=2, b=3.

14) $(x^2 + 1)(x - 1)(x + 1)$
1295=$6^4 - 1 = 37 \times 5 \times 7$

15) o,o,e,o,o,e,o,o,e (repeating o,o,e)
2/3 of the first 999 terms are odd = 666
And the last, so 667

16) $5^{1000} \times 8^{336} = 5^{1000} \times 2^{1008} = 10^{1000} \times 2^8$
256 with 1000 zeros after it. So 1003.

17) Let the number be pq.
$10q + p = \frac{7}{4}(10p + q)$
$q = 2p$
So 48

18) PEF=125
DEP=55
PBQ=55 (alternate segment)
PQB=25+x (x is PEQ)
ABP=x
ABQ=120-x
PBQ=120-2x=55 as above
$x = 32.5°$

19) $a+b>0$
$b+c>0$
$c+d>0$
so (adding)
$a+2b+2c+d>0$

But also:
$a+b+c<0$
$b+c+d<0$
so (adding)
$a+2b+2c+d<0$
Which gives a contradiction

20) $15°$
See solution to question 10 here (with the third being my preferred one):
https://cemc.math.uwaterloo.ca/contests/past_contests/2001/2001EuclidSolution.pdf