Sample Paper 1 Model Solutions

The solutions below are designed to give you an idea of how to approach the questions in the paper and, in some cases, a few alternative approaches. They are not an exhaustive list of possible methods and neither are they necessarily indicative of how much should be written to answer a question fully.

Question 1:

$$\frac{a-b}{c} = \frac{\frac{1}{3} - \left(-\frac{1}{2}\right)}{\frac{1}{4}}$$

$$= \left(\frac{2}{6} + \frac{3}{6}\right) \div \frac{1}{4}$$

$$= \frac{5}{6} \times 4$$

$$= \frac{20}{6}$$

$$= \frac{10}{3}$$

Question 2:

$$\frac{1}{6}(3+4x)$$

$$= \frac{3}{6} + \frac{4}{6}x$$

$$= \frac{1}{2} + \frac{2}{3}x$$

Question 3:

$$5x \div \frac{5}{x}$$

$$= 5x \times \frac{x}{5}$$

$$= \frac{5x^2}{5}$$

$$= x^2$$

Question 4:

$$(x-4)^2$$

$$= (x-4)(x-4)$$

$$= x^2 - 4x - 4x + 16$$

$$= x^2 - 8x + 16$$

Question 5:

$$x^2 + 6x - 7 = (x+7)(x-1)$$

Question 6:

$$(x-3)^2-(x+1)(x-4)$$

$$=x^2-3x-3x+9-(x^2-4x+x-4)$$

$$=x^2-6x+9-(x^2-3x-4)$$

$$=-3x+13$$

Question 7:

$$a(a-b)-b(a-c)-b(c-b)$$

$$=a^2-ab-(ab-bc)-(bc-b^2)$$

$$=a^2-ab-ab+bc-bc+b^2$$

$$=a^2-2ab+b^2$$

$$=(a-b)^2$$

Question 8:

$$\frac{2ab^3}{2a^2b}$$

$$=\frac{ab^3}{a^2b}$$

$$=\frac{b^3}{ab}$$

$$=\frac{b^2}{a}$$

Question 9:

$$\frac{x^2+x}{x^2-1}$$

$$=\frac{x(x+1)}{(x+1)(x-1)}$$

$$=\frac{x}{x-1}$$

Question 10:

$$5+\frac{1}{x}$$

$$=\frac{5x}{x}+\frac{1}{x}$$

$$=\frac{5x+1}{x}$$

Question 11:

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$$

$$1+\frac{a}{b}=\frac{a}{c}$$

$$b+a=\frac{ab}{c}$$

$$bc+ac=ab$$

Question 12:

$$y=\frac{x+1}{x-1}$$

$$y(x-1)=x+1$$

$$xy-y=x+1$$

$$xy-x=y+1$$

$$x(y-1)=y+1$$

$$x=\frac{y+1}{y-1}$$

Question 13:

$$2x+y=7$$
$$3x-2y=5$$
$$4x+2y=14$$ double both sides of top equation
$7x=19$ so $x=\frac{19}{7}$ add second and new first equations
$$2\left(\frac{19}{7}\right)+y=7$$ substitute into first equation
$$38+7y=49$$
$$7y=11$$
$$y=\frac{11}{7}$$
so $\frac{y}{x}=\frac{11}{7}\div\frac{19}{7}=\frac{11}{19}$

Question 14:

$$3-2x\geq 5$$
$$\frac{3}{2}-x\geq\frac{5}{2}$$ halve both sides
$$5-x\geq 6$$ add $\frac{7}{2}$ to both sides

Question 15:

a and b are in the ratio $3:4$ so $b=\frac{4}{3}a$

$$3b=4a$$

$$4a-3b=0$$

Question 16:

$$A(x+2)+B(x-3) \equiv 8x+6$$

$$Ax+2A+Bx-3B \equiv 8x+6$$

This statement must be true for all values of *x* and so the coefficients of *x* have to match, as do the numerical terms. From this we can deduce (then solve) the following simultaneous equations:

$A+B=8$	
$2A-3B=6$	
$3A+3B=24$	treble both sides of first equation
$5A=30$	add second and new first equations
$A=6, B=2$	substitute value of A into first equation
So $A-B=4$	

Question 17:

A: $LHS = 3a+3b = RHS$ so this statement is true

B: $LHS = 3ab = RHS$ so this statement is true

C: $LHS = 3 \times a \times b = 3ab = RHS$ so this statement is true

D: $RHS = 3 \times 3 \times a \times b = 9ab \neq LHS$ so this statement is **false**

Question 18:

A: substitute in $(3,0)$: $3(3)+2(0) = 3 \times 3 = 9$ so this statement is true

B: this equation is equivalent to $2y=-3x+9$

which is equivalent to $y=-\frac{3}{2}x+\frac{9}{2}$ so this statement is **false** (the gradient is $-\frac{3}{2}$)

C: From the above working we can see that the y-intercept is indeed $\frac{9}{2}$. Alternatively, if we substitute in $x=0$, we get that $2y=9$, from which we can deduce that $y=4.5$ and so this statement is true

D: substitute in $(1,3)$: $3(1)+2(3) = 3+6 = 9$ so this statement is true

Question 19:

Method 1: substituting the points into each relation

A: $(a, 0)$: $LHS = a(a) + b(0) = a^2 \neq 0$ so it can't be this one.

B: $(a, 0)$: $LHS = b(a) + a(0) = ab \neq 0$ so it can't be this one.

C: $(a, 0)$: $LHS = b(a) - a(0) = ab = RHS$ so it might be this one.

$(0, b)$: $LHS = b(0) - a(b) = -ab \neq ab$ so it can't be this one.

D: $(a, 0)$: $LHS = b(a) + a(0) = ab = RHS$ so it might be this one.

$(0, b)$: $LHS = b(0) + a(b) = ab = RHS$ so it must be this one.

Alternative approach:

The line passes through $(0, b)$ and so the y-intercept is b.

The gradient is given by $\frac{change\ in\ y}{change\ in\ x}$ between two points so the gradient of this line is $\frac{b-0}{0-a} = -\frac{b}{a}$.

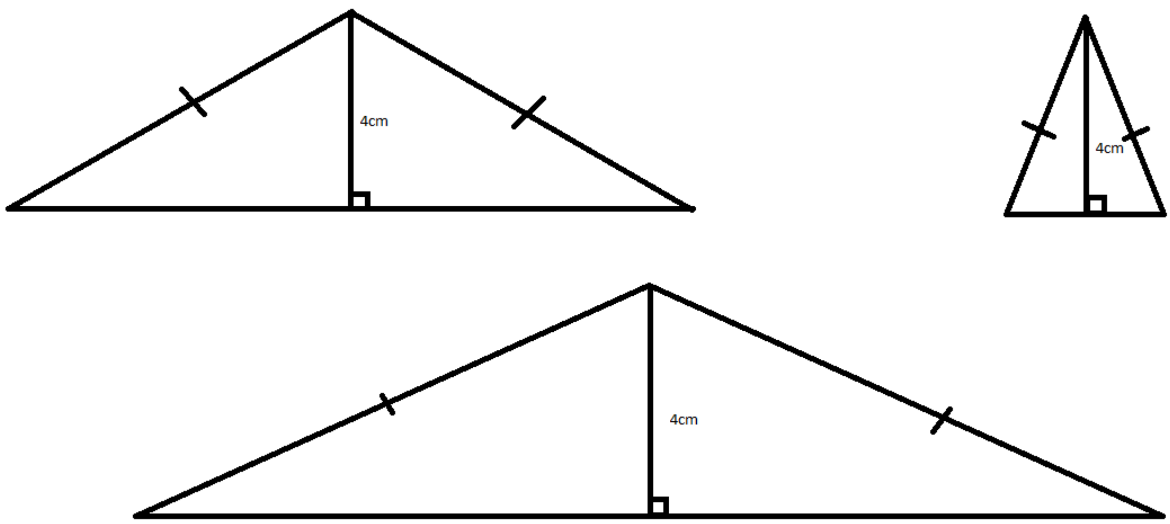
So the equation of the line is $y = -\frac{b}{a}x + b$

which is equivalent to $ay = -bx + ab$

which is equivalent to $bx + ay = ab$

Question 20:

The answer to this is D as there an infinite number of triangles which satisfy the conditions. Some (not to scale) diagrams can be seen below:



Question 21:

Algebraic approach:

The water in the tank forms a cuboid with dimensions $4 \times 4 \times d$, where d is the depth in centimetres.

So we can deduce that $4 \times 4 \times d = 4$

$$4d = 1$$

$$d = \frac{1}{4}$$

Alternative approach:

The total volume is $4^3 = 64$ cm^3. So the container is only $\frac{4}{64} = \frac{1}{16}$ full. So the depth of the water is $\frac{1}{16}$ of 4 which is 0.25cm

Question 22:

To do this, we need the following formulae:

Volume of a sphere $= \frac{4}{3}\pi r^3$ Surface area of a sphere $= 4\pi r^2$

If they are numerically equal we can deduce that:

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\frac{4}{3}\pi r = 4\pi$$

$$\frac{4}{3}r = 4$$

$$r = 3$$

So $V = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi \times 27 = 36\pi$

Question 23:

Let the old price be P and the old volume be V. Therefore, the price per unit volume was $\frac{P}{V}$.

The new price is $1.5P$ and the new volume is $1.2V$.

So the new price per unit volume is $\frac{1.5P}{1.2V} = \frac{1.5}{1.2} \times \frac{P}{V} = \frac{15}{12} \times \frac{P}{V} = 1.25 \times \frac{P}{V}$.

Therefore the price per unit volume has gone up by 25%.

Question 24:

Let Beau's walking speed (in units/min) on Monday be V and the time taken (in minutes) be T. The distance walked is unchanged from Monday to Wednesday so we can deduce that:

$$V \times T = 0.8V(T + M)$$

$$T = 0.8(T + M)$$

$$0.2T = 0.8M$$

$$T = 4M$$

Alternative approach:

She walks at $\frac{4}{5}$ of her original speed and so it will take her $\frac{5}{4}$ of the time. So M represents and extra quarter of the time taken so the time taken must be $4M$.

Question 25:

Algebraic approach:

Let the time Misbah takes to complete the race be t seconds. They cover the same distance so

$$3.8(t + 2) = 4.2t$$

$$3.8t + 7.6 = 4.2t$$

$$7.6 = 0.4t$$

$$t = 19$$

So Misbah takes 19s and Paisley 21s.

In the new race, both will run for 21s so Paisley covers $21 \times 3.8 = 79.8$ m. Misbah will cover $21 \times 4.2 = 88.2$ m. So the head start required is 8.4 m.

Alternative (very easy) approach:

The time taken is actually irrelevant here. Misbah will definitely be running for two extra seconds and so, in that time, can cover $2 \times 4.2 = 8.4$ extra metres.