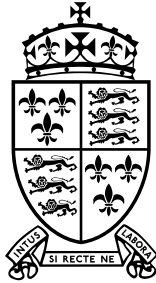


First name:

Surname:

Current School:



**Shrewsbury
School**

Sixth Form Entrance Examination Entry in September 2023

Mathematics (1 Hour 15 Minutes)

Instructions to candidates

Answer all questions, writing your answers in the spaces provided.

The number of marks for each question is shown in square brackets: [].

Section A contains questions of a GCSE nature. Attempt this section first, but do not spend too long on any particular question.

Section B is intended to be considerably more difficult, and is targeted at candidates who are either aiming for an academic scholarship on the strength of their mathematics or hoping to take Further Mathematics at A-Level.

You are expected to use a calculator in this examination.

Relevant working must be shown in order to gain high marks.

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

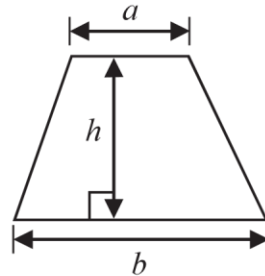
Sum to n terms, $S_n = \frac{n}{2} [2a + (n - 1)d]$

The quadratic equation

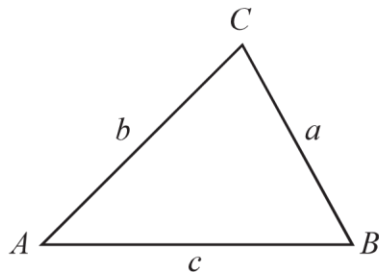
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium = $\frac{1}{2}(a + b)h$



Trigonometry



In any triangle ABC

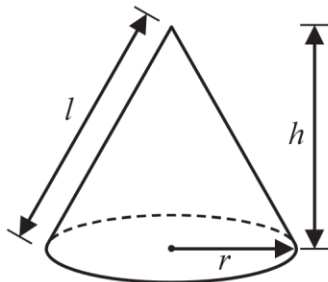
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$

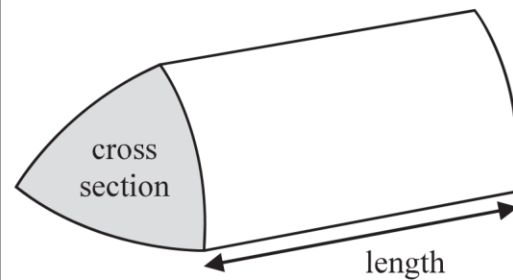
Volume of cone = $\frac{1}{3} \pi r^2 h$

Curved surface area of cone = $\pi r l$



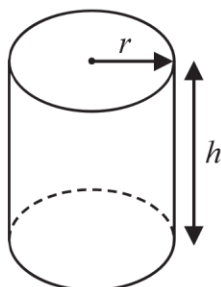
Volume of prism

= area of cross section \times length



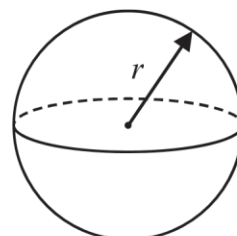
Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi r h$



Volume of sphere = $\frac{4}{3} \pi r^3$

Surface area of sphere = $4\pi r^2$



Section A (60 marks)

Answer all questions in this section.

1) Expand and simplify the following expressions:

a) $5(2x - 4y) - 3(2x - y)$

_____ [2]

b) $(4t - 3)(t - 5)$

_____ [2]

2) Factorise the following expressions fully:

a) $6vw^3 + 9v^3w$

_____ [2]

b) $m^2 - 7m - 30$

_____ [2]

3) You must **not** use a calculator in this question. Full working **must** be shown.

a) Evaluate these, giving your answers as **mixed numbers** in their simplest form:

i) $5\frac{1}{3} - 1\frac{5}{9}$

_____ [2]

ii) $5\frac{1}{3} \div 1\frac{5}{9}$

_____ [3]

b) Simplify the following:

$$4t^2 \times (3t^4)^3$$

_____ [3]

- 4) a) Legolas scores 88% in a tricky Elvish exam. If the exam was out of 150, how many marks did Legolas get?

_____ [2]

- b) A video store is having a Christmas sale, with all prices of Blu-ray discs reduced by 35%. The Blu-ray of the film “The Empire Strikes Back” is on sale for only £8.32. What is the normal selling price of this Blu-ray?

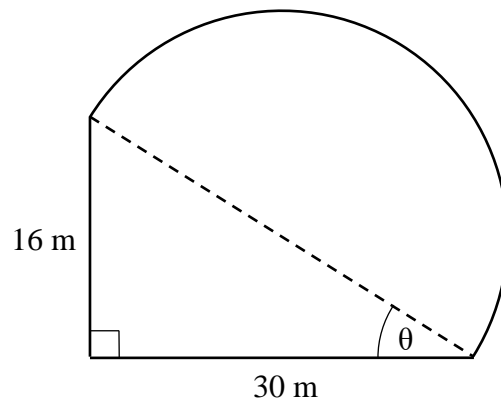
_____ [2]

- c) One Ether was worth \$4,860 in November 2021 and collapsed in value to only \$880 in June 2022.

What was the percentage decrease in the value of one Ether during this period? Give your answer to 3 significant figures.

_____ [2]

- 5) The diagram below shows a garden composed of a right-angled triangular region and a semi-circular region.



- a) Calculate the diameter of the semi-circular region.

_____ [2]

- b) Calculate the total area of the garden to 3 significant figures, giving the correct units in your answer.

_____ [3]

- c) Calculate the perimeter of the garden to 3 significant figures.

_____ [2]

d) Calculate the angle marked θ to 3 significant figures.

_____ [2]

6) The extension of a spring x (measured in cm) is directly proportional to the **square root** of the elastic potential energy E stored inside it (measured in Joules).

When $E = 6.25$, $x = 10$.

a) Find an exact formula for x in terms of E .

_____ [3]

b) Calculate the energy stored in the spring when its extension is 35 cm. Give your answer to 3 significant figures.

_____ [3]

7) Rearrange the following formulae to make t the subject:

a) $2t - 3u = 7$

_____ [2]

b) $m = \frac{6}{t^3}$

_____ [3]

c) $y = 4\sqrt{2t + 1}$

_____ [3]

d) $n = \frac{3-t}{4+t}$

_____ [3]

8) Solve the following equations:

a) $6x + 1 = 2x - 10$

_____ [2]

b) $\frac{4}{x} = \frac{5}{2x-9}$

_____ [3]

c) $2x^2 + 9x - 26 = 0$

_____ [3]

d) $(x^2 + 7)^2 - (x^2 + 5)^2 = 44$

_____ [4]

Spare page for Section A working.

Section B (20 marks)

This section is intended to be considerably more difficult, and is targeted at candidates who are either aiming for an academic scholarship on the strength of their mathematics or hoping to take Further Mathematics at A-Level.

Only attempt these questions if you have done as much of Section A as you can.

B1) a) Expand and simplify $(x + 4)^2$.

_____ [1]

b) Prove that $x^2 + 9x + 17$ is never the square of a whole number.

_____ [3]

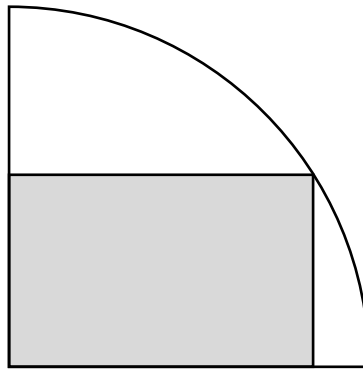
c) Can $x^2 + 9x + 17$ be the cube of a whole number? Explain your answer.

_____ [1]

d) Can $x^2 + 9x + 17$ be the fourth power of a whole number? Explain your answer.

_____ [1]

- B2) The diagram below shows a shaded rectangle inscribed in a quarter-circle. The long and short edges of the rectangle are in the ratio 3:2.



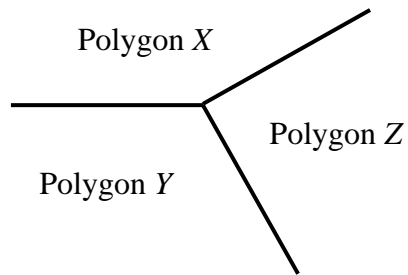
Find an exact expression for the proportion of the quarter-circle that is shaded, giving your answer in terms of π .

_____ [6]

Please turn over for the final question.

B3) Three regular polygons meet at a common vertex as shown below.

The polygons have x , y and z edges, all of unit length, where $x \geq y \geq z \geq 3$.



a) Prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = k$ where k is a constant to be determined.

_____ [3]

b) Find the largest possible value of x when $z = 3$.

_____ [2]

c) Find the largest possible value of x when $z = 4$.

_____ [1]

d) Find the largest possible value of x overall, giving a reason for your answer.

_____ [2]

+++++ **END** +++++

Spare page for Section B working.