You may not use a calculator in this paper.

All working should be clearly shown.

You should attempt as many questions as possible, in any order you like.

1   a   Fifteen hives of bees can make 50 jars of honey each week.
          How many jars of honey could twenty-one hives of bees make each week?
     b   Fifteen cows take 12 weeks to produce 10000 litres of milk.
          How many weeks would twenty cows take to produce 10000 litres of milk?

2   In each part of this question, find three different numbers which add up to a total of 60 and which have the properties stated.

     a   The middle number is equal to the average of the smallest and the largest numbers.
          The difference between the largest number and the smallest number is 12.

     b   The smallest number is half the average of the larger two numbers.
          The largest number is twice the average of the smaller two numbers.

3   A school is divided in the ratio 3 : 2 between seniors and juniors.
     Of the seniors, $\frac{7}{12}$ like rice pudding. Of the juniors, $\frac{3}{8}$ like rice pudding.
     What fraction of the school likes rice pudding?

4   Peter has £24 in his account. He starts to pay £2·20 a week into this account.
     Simon has £35 in his account. He starts to pay £1·50 a week into this account.
     After how many weeks will Peter first have more in his account than Simon?

5   Work out the following. Give your answers as exact fractions in their lowest terms.

     a $\dfrac{879978789}{999999999} - \dfrac{213312123}{999999999}$

     b $\dfrac{241213423}{999999999} \div \dfrac{482426846}{333333333}$

     c $\dfrac{50000000}{222222222} + \dfrac{225000000}{333333333}$

6   A recipe for a large pie requires one and a quarter pounds more flour than butter.
     Flour costs 56 pence per pound and butter 66 pence per pound.
     The cost of buying flour and butter to make the pie is £7·41.

     How many pounds of butter does the recipe require?

7 In the diagram, AB is equal to AC and BX is equal to AX.
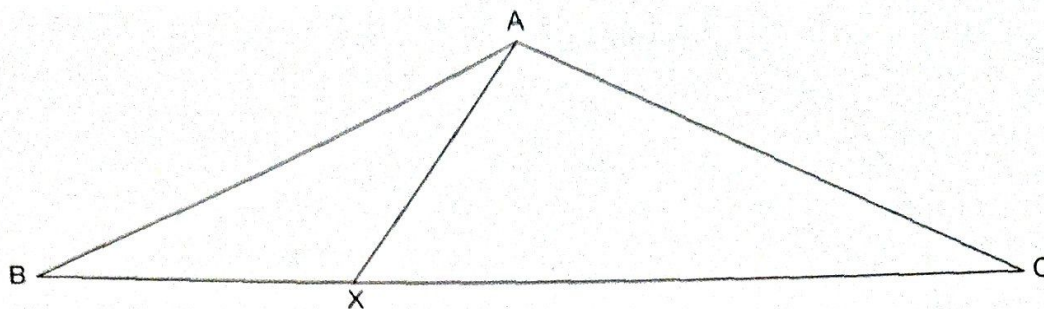
Angle ABX = 32°.
Find angle CAX, giving and explaining each step in your working.

8 **a** **i** Multiply out the brackets in $(x+3)^2 + (x-3)^2$.

   ii Simplify $\dfrac{(x+3)^2 + (x-3)^2}{2}$.

   iii Show how to use your result in part **ii** to find two square numbers whose average is 109.
   iv Find two square numbers whose average is 1 000 001?

   b **i** Factorise $x^2 + x - 12$
   ii Show how to use your result in part **i** to find the value of 1004×997.
   iii Find the value of 103×97.

9 The diagram shows a rectangle ABCD. A triangle APQ is inscribed in the rectangle.
The lengths AB = 18 cm, AQ = 15 cm, DQ = 12 cm and CP = 8 cm are shown in the figure .
   a Find the lengths of PQ and PB.
   **b** Show that triangle APQ is right-angled.
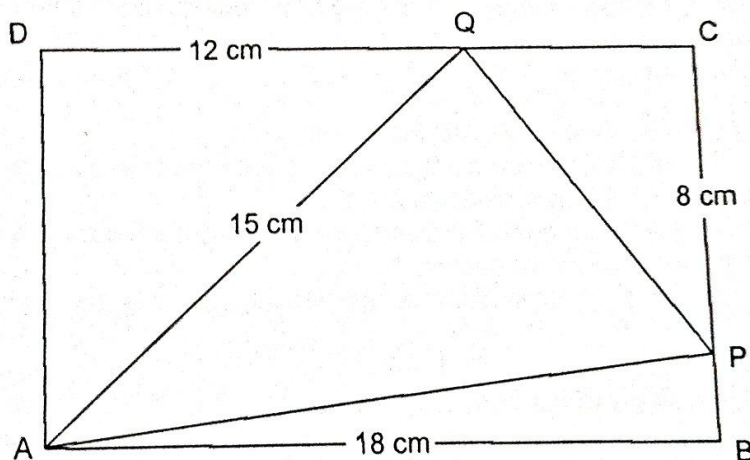
10 In a football league, each game can be won, drawn or lost.
A team gets 3 points for a win, 1 point for a draw and 0 points for a loss.
   a Explain why it is not possible for a team to play 8 games and score 23 points.
   b Explain why it is not possible for a team to play 13 games without losing and score 28 points.
   c **i** A team has played four games and scored 9 points. How many wins, draws and losses must this team have had?
   ii A team has played four games and scored 6 points. Explain why it is not possible to tell for certain how many wins, draws and losses this team had.
   iii A team has played six games. What is the largest number of points the team can have scored if it is not possible to tell for certain how many wins, draws and losses the team had?

11 The diagram shows three circles, **X**, **Y** and **Z**. All their diameters lie along the line AB.
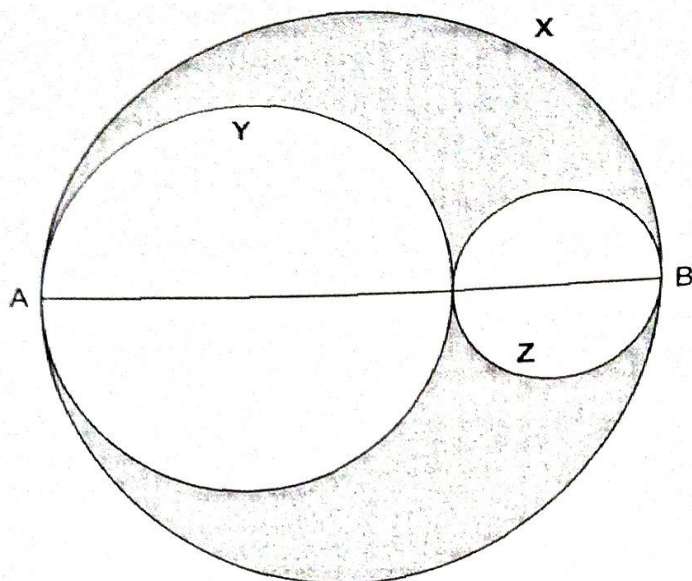
The radius of circle **Y** is y cm and the radius of circle **Z** is z cm.

a What is the radius of circle **X**?

b Find the fraction $\dfrac{\text{area of circle Y + area of circle Z}}{\text{area of circle X}}$ in terms of y and z.

c If the radius of circle **Y** is twice the radius of circle **Z**, what fraction of circle **X** is shaded?

12 In the game of digits, two players take it in turn to write down one of the digits 1, 2, 3, 4.

After each turn, look at the last two digits that have been written down. If these are $m\,n$ and the number of times digit n occurs in the complete list is m, then the last player has won. If the last player has not won, the game continues.

So player A might start with 4 and player B follow with 3.

    4 3   this is not a win for B because 3 does not appear 4 times in the list.

so play continues: player A might write down 2

    4 3 2   this is not a win for A because 2 does not appear 3 times in the list.

so play continues: player B might write down 4

    4 3 2 4   this is a win for B because 4 does appear 2 times in the list.

a Explain why A should not start with 1 or 2.

b Explain why, if A starts with 3 or 4, B should not write 1 or 2.

c How should B respond if A starts with 3? Explain your answer.

d Explain how B can win if A starts with 4.

e Explain how A can always win.