Westminster School Challenge 2005

Mathematics II

1½ hours

You are expected to use a calculator in this paper.

All working should be clearly shown.

You should attempt all the questions, in any order you like.

1 103 ounces is the same weight as 2920 grams.
How many ounce weights can be made from 1000 kg of brass?

2 Sam travels from Arundel to Cheltenham via Birmingham. It takes him 4 hours and 20 minutes to travel from Arundel to Birmingham at an average speed of 45 kilometres per hour. He then travels the 65 kilometres from Birmingham to Cheltenham at an average speed of 25 kilometres per hour.
What is Sam's average speed over the whole journey from Arundel to Cheltenham?

3 a Simplify $\frac{2}{3}(1-2x)-\frac{4}{3}x+\frac{7}{3}$.

b Solve the equation $\frac{2}{3}(1-2x)-\frac{4}{3}x+\frac{7}{3}=11$

c What is the result of multiplying $1+\frac{2}{x}$ by x?

d Solve the equation $\frac{2}{x}=\frac{x}{5}$.

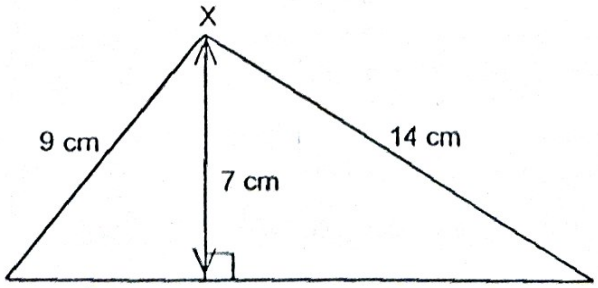
4 "Mach numbers" are a way of giving speeds. A speed of 330 metres per second is called "Mach 1"; "Mach 2" corresponds to a speed of 660 metres per second, and so on.
The moon is 384 500 kilometres from the Earth. A space craft left Earth at breakfast time (8 a.m.) and arrived at the moon in time for tea (4 p.m.). At what average speed did the space craft have to fly?
Give your answer as a Mach number.

5 A ski lift, which runs continuously all day, every day, takes exactly 42 minutes to get from its base at the foot of a mountain to the top of the mountain and back again to its base.
One Monday, the ski lift leaves its base for the first time at exactly 00:00 hours, so it leaves its base for the second time that day at exactly 00:42.

a What is the last time that day at which the ski lift leaves its base?

b What day of the week is it when the ski lift next leaves its base at exactly 00:00 hours?

6 Find the area of the triangle shown.
Angle X is *not* a right angle.



7 When n is a positive integer, the value of A is calculated using the formula $A = n(13000 - n^3)$.

a Show that A takes the same value if $n = 4$ or 22.

b Use your calculator to determine which positive integer value of n makes A as large as possible.

8 a Eastbourne advertises its delights by claiming "28% less annual rainfall than Scarborough". If the claim is true and Eastbourne has 245 mm less annual rainfall than Scarborough, calculate the annual rainfall of each town.

b Glasgow advertises its delights by claiming "36% greater average annual porridge consumption per person than Aberdeen". If the claim is true and Glaswegians eat on average 238 kg of porridge annually, what weight of porridge do Aberdonians eat on average each year?

c A DVD normally costs £23·50, *including* VAT at 17·5%.
A company in Jersey offers a 32% reduction on the price, *excluding* VAT, of all DVD's.
If the reduced price is less than £18, VAT no longer has to be paid at all.

i At what price do the company in Jersey offer the DVD?

ii What percentage discount can the company advertise on the full price including VAT?

9 Solve the simultaneous equations

$y = 4x - 5$

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10 The following small denomination British stamps are available:

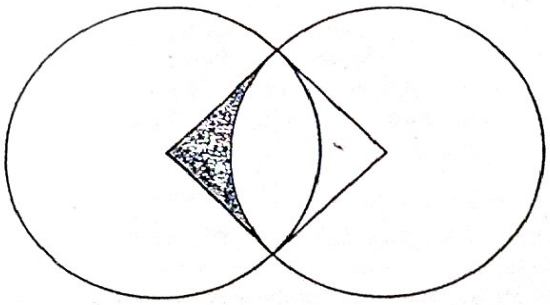
1p, 2p, 5p, 7p, 10p, 20p, 34p, 35p, 39p, 40p, 42p, 43p, 47p, 50p, 68p.

Tom has exactly three small denomination British stamps. The sum of their values is £1·20.
Write down all the different sets of stamps he might have.

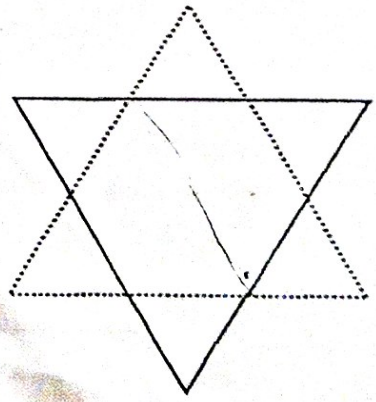
11 a The diagram shows a square and two circles, whose centres are at opposite corners of the square. The radius of each circle is 2 cm and the side of the square is also 2 cm in length.

i What is the shaded area?

ii What is the area of that part of the square which is in *both* circles?



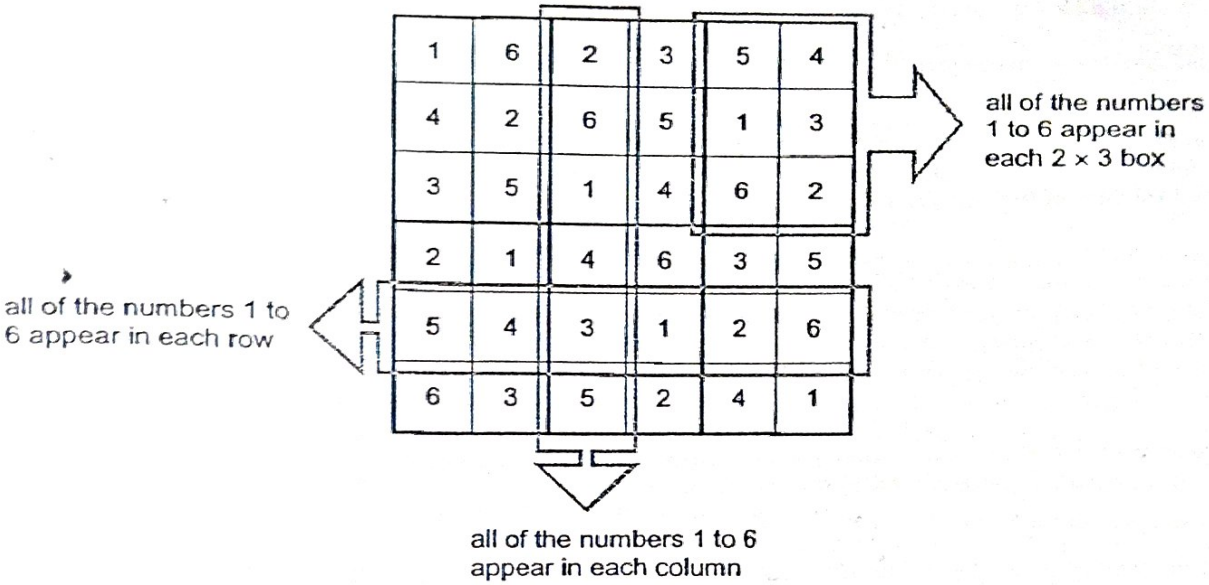
b The diagram shows two large equilateral triangles, one with a solid line, the other with a dotted line. They each have side length 6 cm. They are placed symmetrically so that the area that is in *both* triangles forms a regular hexagon.
Find the area of the regular hexagon.



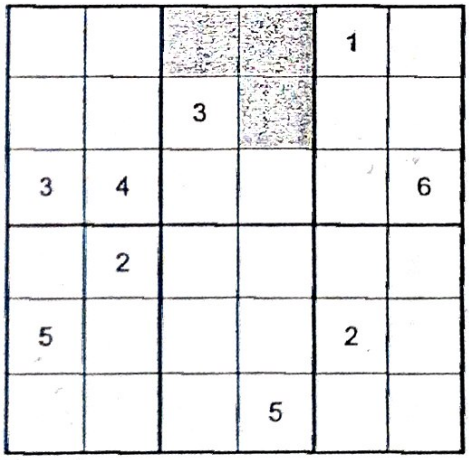
12 In a type of Su-Doku puzzle, each of the thirty-six squares must be filled with a whole number from 1 to 6 inclusive. The numbers 1 to 6 inclusive must each appear exactly once:

- in each row
- in each column
- in each of the six 2 × 3 boxes marked by heavy lines.

The diagram below shows a completed puzzle.



The diagram below shows a puzzle with some of the numbers entered.



a Explain why the number 3 *must* be entered in the top right hand corner of the puzzle.

b Copy and complete the top right hand 2 × 3 box, explaining how you can be *sure* that each of your entries is correct.

c Explain why the number 2 *must* be entered in the top left hand corner of the puzzle.

d What numbers *must* be entered in the three shaded squares? Explain how you can be *sure* that each of your entries is correct.

e IF YOU HAVE DONE ALL YOU CAN OF THE REST OF THIS PAPER, try to complete the puzzle.