Westminster School Challenge 2005

Mathematics III

1½ hours

You may not use a calculator in this paper.

All working should be clearly shown.

You should attempt as many questions as possible, in any order you like.

1 Two fifths of Tom's birthday cake is taken by the Head Master. Three quarters of the remainder is taken by the other boys in his form.
What fraction of his cake is Tom left with?

2 It costs £60 a year to join Westcott Swimming Club. Members pay £2.40 each time they use the Westcott pool. Non-members pay £3.50 each time they use the pool.
Toby goes swimming once a week in the Westcott pool. Would it save him money to join the Swimming Club? Show clearly the calculation you do to decide.

3 A swarm of 3 million locusts takes 40 minutes to devastate 25 hectares of maize. How long would a swarm of 8 million locusts take to devastate 30 hectares of maize?

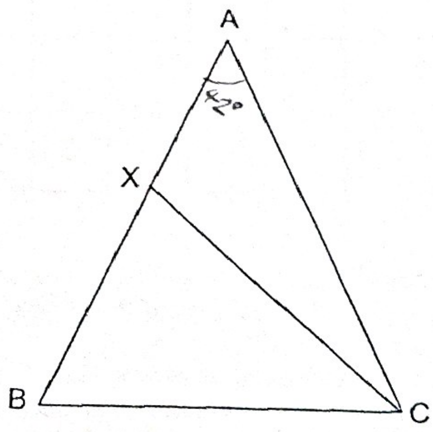
4 The number 555 can be written as the sum of the squares of three different prime numbers.
$555 = 19^2 + 13^2 + 5^2$.

a Explain why, if an even number can be written as the sum of the squares of three different prime numbers, then 2 must be one of the prime numbers.

b Write 222 as the sum of the squares of three different prime numbers.

5 One hundred and seven people get together in teams to take part in a quiz. Each team has three, four or five members.
There are twenty-seven teams altogether and there are twice as many teams with four members as there are teams with three members.
How many teams have five members?

6 In the diagram, AB = AC and CX = CB.



Angle BAC = 42°.
Find angle ACX, giving and explaining each step in your working.

7 The positive whole numbers are written into a grid which is six squares wide, as shown. In each new row the order of the numbers changes direction. The fifth column of the grid is shaded.

1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
24	23	22	21	20	19

If the grid and the numbering carry on in the same way,

a which number will appear in the fifth column of the eightieth row;

b where will the number 1000 appear?

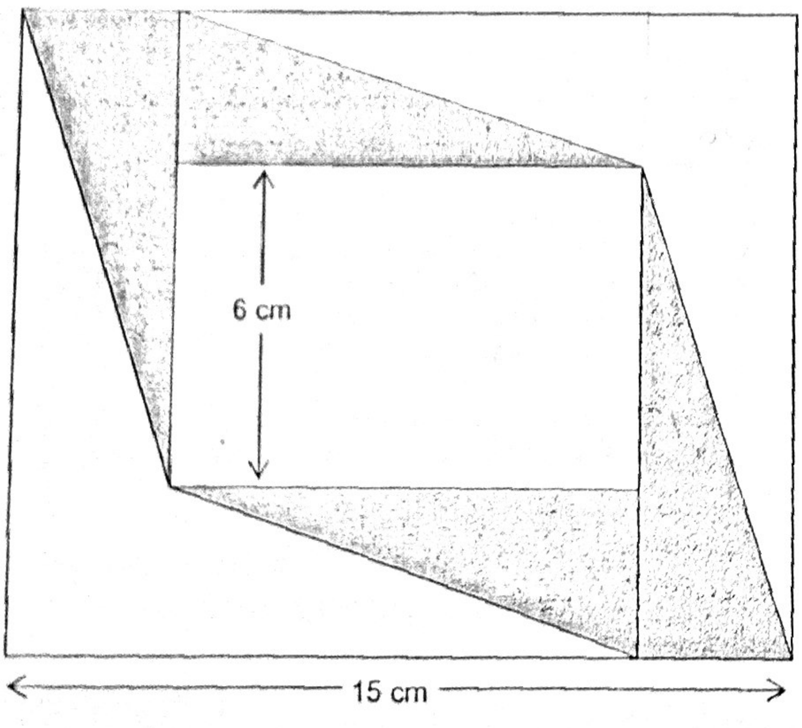
8 Suppose you toss a coin ten times and write down the results in order as a list e.g. HHTHHHTHTT.

Discuss the arguments in each of **a** and **b**. In each case, you should say whether statement **i** is true and whether statement **ii** follows from statement **i**. Explain your reasons carefully.

a i The probability of HHHHHHHHHH is equal to the probability of HTHTHTHTHT,
ii *therefore*, it is equally likely that you will get 5 heads and 5 tails or ten heads.

b i It is very unlikely that you will get HHHHHHHHHT,
ii *therefore*, if you get nine heads in a row, your tenth toss is very likely to show a head.

9 The four shaded, right-angled, triangles in the diagram are identical. They are drawn in an outer rectangle of length 15 cm and they form an inner rectangle of width 6 cm.
What percentage of the outer rectangle is shaded?
Show clearly and explain the calculations you do to decide.



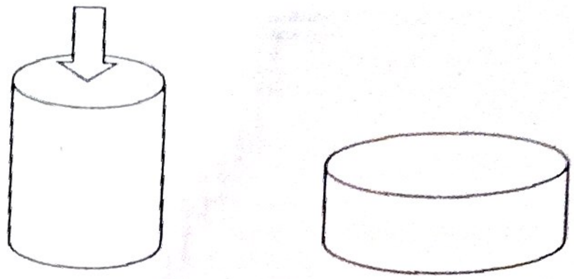
10 a i Multiply out the brackets in $(3x+4)^2+(4x-3)^2$.
ii Show how to use this result to find two square numbers whose sum is 1625.

b i Factorise x^2-9
ii Show how to use this result to find the prime factors of 9991.

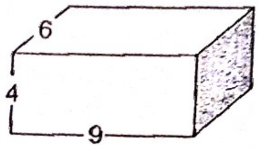
11 a I have a box of chocolate covered nuts. Two thirds of them are hazel nuts and the rest macadamias. Half of the nuts are covered in milk chocolate and half in plain chocolate. If three quarters of the macadamias are covered in plain chocolate, what fraction of the hazel nuts are covered in plain chocolate?

b Originally, half of a box of my favourite chocolates were truffles. After I ate some of the truffles, a quarter of the remaining chocolates were truffles. What fraction of the truffles did I eat?

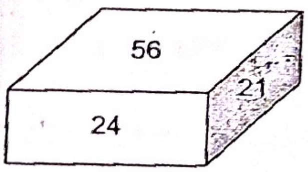
12 A rubber cylinder of height 18 cm and radius 3 cm is squashed so that it remains a cylinder of the same volume, but has height 8 cm. What is its new radius?



13 a A cuboid has dimensions: length 9 cm, width 6 cm and height 4 cm. What are the areas of the faces of the cuboid?



b The areas of the faces of a cuboid are 56 cm^2, 24 cm^2 and 21 cm^2. What are the dimensions of the cuboid?



c Calculate $\frac{21\times 24}{56}$, $\frac{56\times 21}{24}$ and $\frac{24\times 56}{21}$.

d How are your answers to **b** and **c** related?

e Explain carefully why this relationship will always hold, whatever the dimensions of the cuboid.

f The areas of the faces of a cuboid are 4 cm^2, 6 cm^2 and 12 cm^2. What are the dimensions of the cuboid? The answers are not whole numbers.

14 If you are asked to apply RULE(2, 3) to a number, you should first multiply the number by 2 and then add 3 to the result. If you are asked to apply RULE($\frac{2}{3}$, $-\frac{1}{2}$) to a number you should first multiply the number by $\frac{2}{3}$ and then subtract $\frac{1}{2}$ from the result.

a When RULE ($\frac{2}{3}$, −1) is applied to a number, the result is 5. What was the number?

b What *single* rule always has the same effect as applying RULE($\frac{2}{3}$, −2) followed by RULE(3, 6)?

c Applying RULE(2, 3) followed by RULE(3, 2) to a number does not give the same result as applying RULE(3, 2) followed by RULE(2, 3) to the number. Show that the difference between the answers is always the same.

d Find the value of *n* so that applying RULE(2, *n*) followed by RULE(3, 2) to a number gives the same result as applying RULE(3, 2) followed by RULE(2, *n*) to the number.

e Find the values of *a* and *b* so that applying RULE(2, 3) followed by RULE(*a*, *b*) to a number always gives the original number.