Name

Westminster School Challenge 2007

Mathematics III

1½ hours

You may not use a calculator in this paper.

All working should be clearly shown.

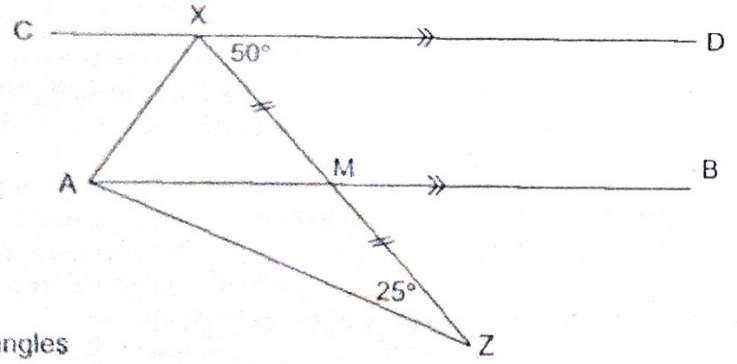
You should attempt as many questions as possible, in any order you like.

1 There are two-spotted ladybirds and seven-spotted ladybirds. The ladybirds in my coccinellarium have a total of 103 spots. I have more seven spotted than two-spotted ladybirds.
How many of each do I have?

2 With a 42 kilogram barrel of marmalade, I can feed eight Peruvian bears for a week *or* twelve Columbian bears for a week. I have twelve Peruvian bears *and* eight Columbian bears. How many kilograms of marmalade do I need to feed them for a week?

3 The members of a sports club are all either swimmers and squash players: $\frac{2}{5}$ of the club are swimmers. If $\frac{5}{8}$ of the swimmers and $\frac{3}{4}$ of the squash players use the facilities every day, what fraction of the club does not use the facilities every day?

4 In the diagram line CD is parallel to line AB, and M is the midpoint of line XZ.
Angle DXM = 50° and angle MZA = 25°.



Find angles
i MAZ
ii CXA.
showing and explaining each step in your working.

5 Solve these simultaneous equations.

$a + b - c = 3$,
$c + a - b = 10$,
$b + c - a = 20$.

6 Of the 120 boys in a prep school, 53 are neither in the orchestra nor belong to the chess club.
The number of boys who belong to the chess club is 12 more than the number that is in the orchestra.
One third of the boys who are in the orchestra also belong to the chess club.
How many boys are in the orchestra and how many belong to the chess club?

7 **In this addition sum, each of the digits 1 to 9 is used once each.**
However, six of the digits have been replaced by letters.

$$\begin{array}{ccc} 3 & A & 2 \\ B & 8 & C \\ \hline D & E & F \end{array}$$

a Explain why B can only be 1, 4, 5, or 6.

b Explain carefully why, if B is 4, then A must be 1.

c Show carefully that if you choose B to be 4 or 6, it is not possible to complete the sum correctly.

d Find all the possible ways of choosing A, B, C, D, E and F so that the sum is completed correctly.
Show carefully that you have found all the ways.

8 To make orange squash, you mix orange concentrate with water. Kind Mrs King mixes her squash in the ratio of one part concentrate to five parts water. Mean Mrs Martin mixes her squash in the ratio of one part concentrate to nine parts water.

a What fraction of Mrs King's squash is concentrate?

b If 3 litres of Mrs King's squash and 7 litres of Mrs Martin's squash were mixed, what fraction of the mixture would be concentrate?

c When x litres of Mrs King's squash and 5 litres of Mrs Martin's squash were mixed, $\frac{1}{6}$ of the mixture was concentrate. What is the value of x?

9 Ian donates 10% of his salary each month to charity.
Jon donates 20% of his salary each month to charity.
The amount Jon donates is 50% more than the amount Ian donates.
What percentage increase in Jon's salary would make it equal to Ian's salary?

10 To **double** a number means to multiply it by two. In this question, to **twiddle** a number will mean to add four to it, and to **flip** a number will mean to subtract it from 8 (so flipping 3 gives 5, flipping −1 gives 9 and so on).

a If you **twiddle** a number and then **twiddle** the answer, the overall effect of the two operations is to add eight to the number. What would be the overall effect of the two operations if you:
   i **twiddle** a number and then **twiddle** the answer?
   ii **twiddle** a number and then **flip** the answer?
   iii **flip** a number and then **twiddle** the answer?

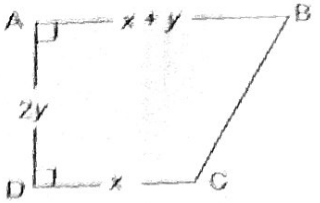
b Show that if you **twiddle** a number, **flip** the answer and then **twiddle** the answer to that, this has the same overall effect as just **flipping** the number once.

c Find a sequence of three operations, each a **twiddle** or a **flip**, that you can carry out on any number and which has the overall effect of changing the sign of the number.

d Can you find a sequence of operations, each a **twiddle** or a **flip**, that has the overall effect of leaving every number you apply it to unchanged, but which does not consist only of **flips**?

11 a Multiply out the brackets in $(x + y)^2 - x^2$ and simplify your answer.

b The diagram shows a quadrilateral ABCD.
Side DC has length x cm,
side AB has length $x + y$ cm,
side AD has length $2y$ cm
angles BAD and CDA are right angles.



Write an expression for the area of the quadrilateral in terms of x and y.

c In fact, the area of the quadrilateral is 72 square centimetres, and side $AB = x + y = 11$ cm.
Find the values of x and y.

12 a The simultaneous equations below have the solution $x = 5$, $y = -3$.

$$2x - 7y = \bigcirc$$
$$\square x + 3y = 11$$

Find the numbers which must be under the square and the circle.

b The simultaneous equations below have the solution $x = 5$, $y = -3$.

$$\bigcirc x - \square y = 53$$
$$\square x + \bigcirc y = 9$$

Find the numbers which must be under the square and the circle (the *same* number must be in *both* squares, and the *same* number in *both* circles).

c The simultaneous equations below have the solution $x = 5$, $y = -3$.

$$5x - 2y = 31$$
$$\square x + \bigcirc y = 1$$

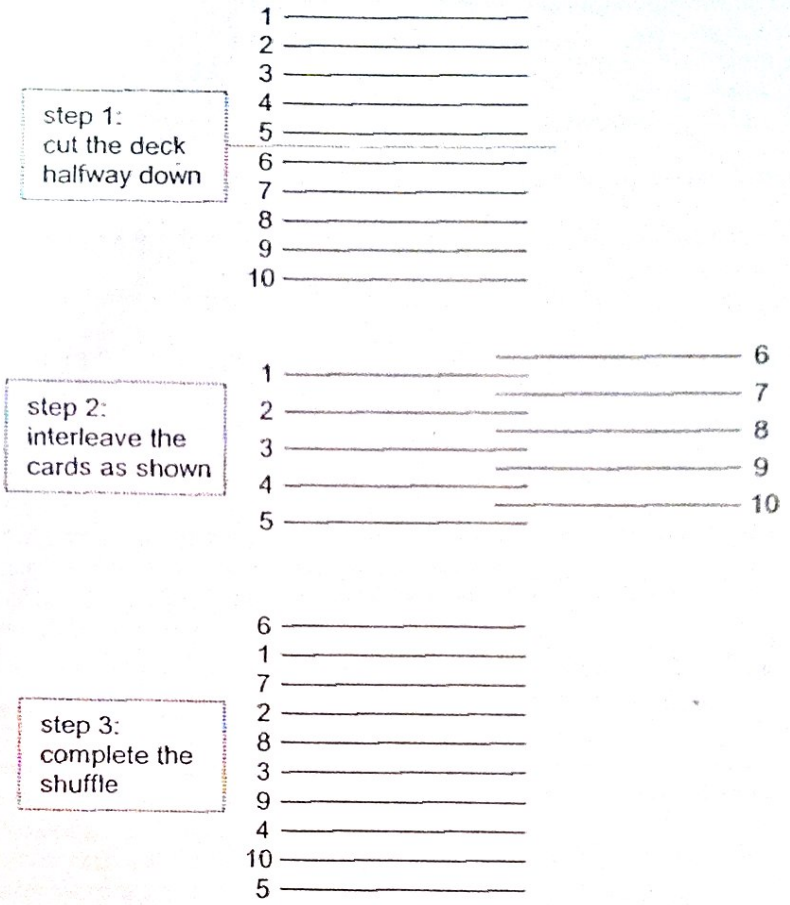
i Find two *whole* numbers which *could* be under the square and the circle.
ii How many different possible pairs of *whole* numbers could be under the square and the circle? Can you show how to get *all* the possibilities?

13 a If you pulled three sweets, one after another, at random out of a bag containing some toffees and some chocolates, which of these events would have the largest probability and which would have the smallest probability? Explain your answers carefully.
i at least one is a toffee;
ii the first one is a toffee and the other two are not;
iii exactly one of the three is a toffee.

b My class has equal numbers of boys and girls in. Almost all the boys and hardly any of the girls like watching football. If I had one ticket for a football match, and gave it out at random to one person in my class, which of these events would have the largest probability and which would have the smallest probability? Explain your answers carefully.
i I give the ticket to a boy;
ii I give the ticket to a boy who does not like watching football, or to a girl;
iii I give the ticket to a boy who likes watching football.

Turn over for the last question.

14 The picture shows how a perfect riffle in-shuffle of an even number of cards is carried out.
In the example, there are 10 cards in the deck, numbered 1 to 10. At the start, the cards are in order.



Notice that the lower half of the deck is shuffled in *above* the upper half of the deck.

a A second perfect riffle in-shuffle is carried out on the ten cards shown in the diagrams above.
Show the order of the cards after this second shuffle.

b A perfect riffle in-shuffle is carried out on a deck of two hundred cards numbered 1 to 200, starting in numerical order. **You should not attempt to draw a diagram of this shuffle.**
After the shuffle
- i what number is on the fifteenth card counting from the top of the deck?
- ii what number is on the forty-fourth card counting from the top of the deck?
- iii where is the card numbered 27?
- iv where is the card numbered 158?
- v if n is even, what number is on the n^{th} card counting from the top of the deck?
- vi if n is less than or equal to 100, where is the card numbered n?
- vii if n is odd, what number is on the n^{th} card counting from the top of the deck?
- viii if n is more than 100, where is the card numbered n?

c The problem with perfect riffle in-shuffles is that they are too perfect: after a certain number of such shuffles have been carried out on a deck, the deck will be back in its original order!
With a deck of fourteen cards numbered 1 to 14, starting in numerical order, find how many perfect riffle in-shuffles are required to return the deck to its original order.