Westminster School Challenge 2009

Mathematics II

1½ hours

You are expected to use a calculator in this paper.

All working should be clearly shown.

You should attempt all the questions, in any order you like.

1 A teacher takes £100 on a school trip. He has to buy train tickets with this money. His ticket costs £14·74, and each pupil's ticket costs £6·36. He has enough money for the tickets he needs, but he realises that if one more pupil had turned up for the trip, he would not have had enough money.

a How many pupils did he take with him on the trip?

b How much change did he get?

2 At 11:25, a train leaves Kings Cross Station for Edinburgh.
The train travels non-stop from Kings Cross to York, which is a distance of 328·7 kilometres.

a If the train arrives at York at 13:19, find the average speed of the train.

The train leaves York station at 13:24. The distance from York to Edinburgh is 304·2 kilometres.

b If the average speed of the train in the second part of the journey is 108 kilometres per hour, find the time at which the train arrives in Edinburgh.

3 **a** Simplify $x - 2(x - 1) - 1$.

b What is the result of dividing $6a$ by $3a$?

c By what would you have to multiply $2x$ to get x^2?

d Solve the equation $\frac{x+8}{2} = 2x + 4$.

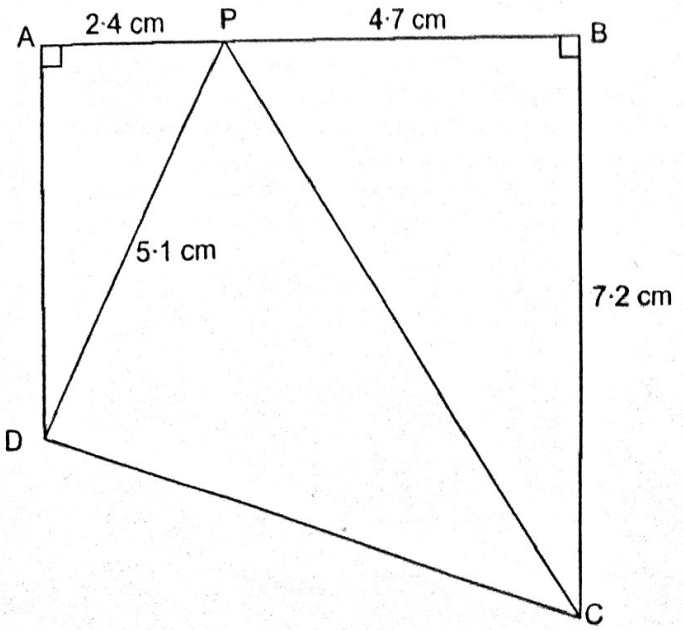
4 In the diagram, ABCD is a trapezium with right angles at A and B.
BC = 7·2 cm, PB = 4·7 cm, AP = 2·4 cm and PD = 5·1 cm.

a Find length PC.

b Find length AD.

c Find the area of trapezium ABCD.

d Find the area of triangle PCD. Note that this triangle is **not** right-angled.



5 **a** The annual interest rate paid by my bank to its savers has fallen from 6% last year to 0·6% this year. I have the same amount in my saving account this year as last year, but this year I will get £668·25 less interest. How much do I have in my savings account?

b My house rose in value by 13% in 2006 and by 8% in 2007.

i By what overall percentage did my house rise in value between the start of 2006 and the end of 2007?

Unfortunately, by the end of 2008, my house was worth exactly the same as at the start of 2006.

ii By what percentage did the value of my house fall in 2008?

6 **a** Find 126^2.

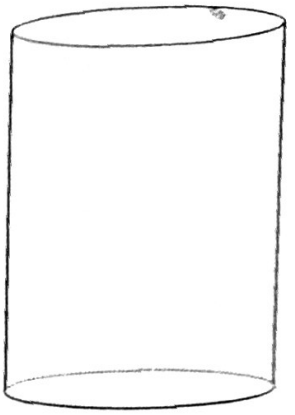
b Show that no whole number between 15900 and 16100 is the square of a whole number.

c Show that no whole number between 240000 and 250000 is the cube of a whole number.

d Find three consecutive whole numbers which multiply together to make a whole number between 240000 and 250000. [Consecutive whole numbers are next to each other in order, e.g. 11, 12, 13.]

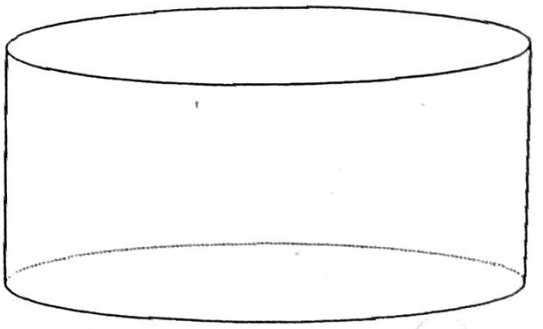
7 The first diagram shows a solid cylinder with height 15·1 cm and radius 4·8 cm.

a Find the volume of the cylinder.



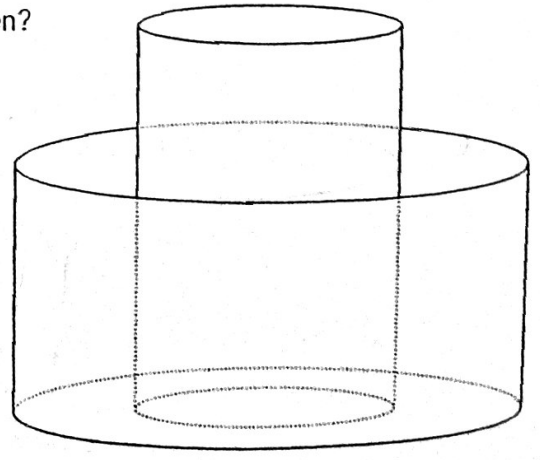
The second diagram shows a hollow cylindrical bowl with height 10 cm and internal radius 9 cm. 1120 cm^3 of water is poured into the bowl.

b Find the depth of water in the bowl.



The third diagram shows how the solid cylinder has been placed in the bowl.
The amount of water in the bowl stays the same.

c By how much has the water level in the bowl risen?



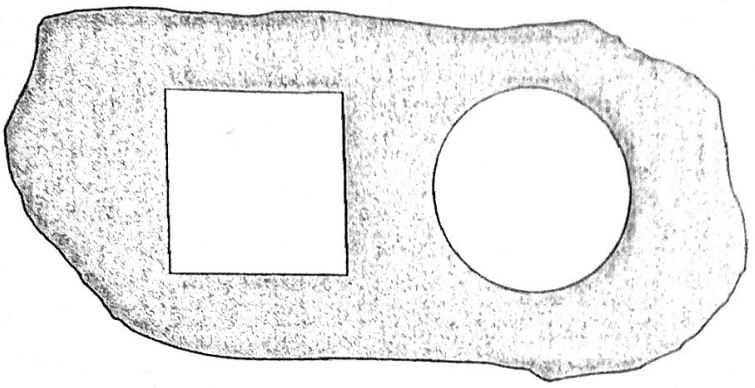
8 Robert promised to eat five pieces of fruit a day on every day in April (there are 30 days in April).
In fact, he did not keep his promise, but he did manage to eat three or more pieces of fruit on every day in April, and he ate five pieces of fruit on twice as many days as he ate four pieces of fruit.
He ate a total of 125 pieces of fruit during April.
Let x be the number of days on which he ate four pieces of fruit.
Write an equation for x and solve it.

9 In the obscure country of Philatelia, large and small letters cost different amounts to send. Stanley Gibbon, a resident of Philatelia, observes the following.

- If he buys books of 14¢ stamps, he can send either large or small letters using a whole number of these stamps, though even small letters cost more than 14¢ to send.
- One day, he sends several large letters and spends 210¢; the next day he sends several small letters and spends 210¢.

Given that a small letter and a large letter cost different amounts to send, how much does each size of letter cost to send?

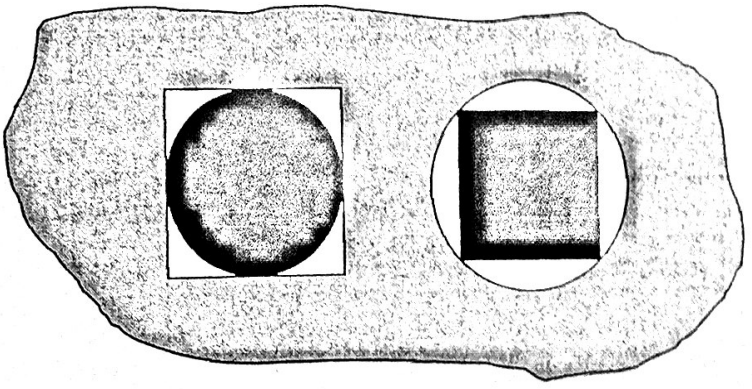
10 The diagram shows two holes. One is square and the other is round (circular).



The square has side length 3·9 cm. The square and the circle have the same area.

a Find the radius of the circle.

The diagram shows the same two holes. The square hole has a round peg in it. The round hole has a square peg in it. The pegs touch the sides of the holes.



b What percentage of the area of the square hole is occupied by its peg?

c What percentage of the area of the round hole is occupied by its peg?