Westminster School Challenge 2010

Mathematics III

1½ hours

You may not use a calculator in this paper.

All working should be clearly shown.

You should attempt as many questions as possible, in any order you like.

1 I like German and Italian opera. I own 325 opera recordings: 135 German opera recordings and 190 Italian opera recordings. Of my 325 opera recordings, 255 are on CD and 70 are on DVD.
I have 110 German opera recordings on CD. How many Italian opera recordings do I have on DVD?

2 Solve the simultaneous equations

$x + 2y = -5$

$x - 2y = 12$

3 I have 18 tortoises. With 28 kg of tortoise feed I can feed my tortoises for 12 weeks.
I acquire two more tortoises.
For how many weeks can I feed my 20 tortoises with 35 kg of tortoise feed?

4 The **harmonic average** h of two numbers a and b is given by the formula

$$h = \frac{1}{\frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)}$$

So the harmonic average of 3 and 7 is

$$h = \frac{1}{\frac{1}{2}\left(\frac{1}{3} + \frac{1}{7}\right)} = \frac{1}{\frac{1}{2}\left(\frac{10}{21}\right)} = \frac{1}{\frac{5}{21}} = \frac{21}{5} = 4\frac{1}{5}$$

a What are the harmonic averages of

i 6 and 10,

ii $7\frac{1}{2}$ and 30.

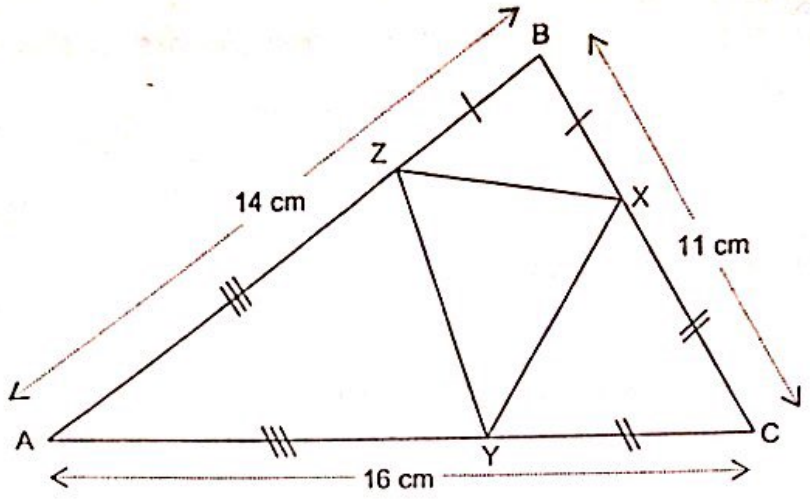
b The harmonic average of two numbers is 5. One of the numbers is 3. What is the other?

5 Thirty percent of the pupils at Miss Twinkerton's Academy are boys; sixty-five percent of the pupils at Mr Squeers' Preparatory are boys. When Miss Twinkerton becomes Mrs Squeers, the two schools amalgamate: fifty percent of the pupils are then boys.
Before amalgamation, there were 360 pupils at Mr Squeers' Preparatory. How many were there at Miss Twinkerton's Academy?

6 **a** When I ask them, sixty percent of my Year 9 Mathematics set say they enjoy mathematics. Give two reasons why it would not be sensible to estimate the proportion of the whole of Year 9 who enjoy mathematics as 60%.

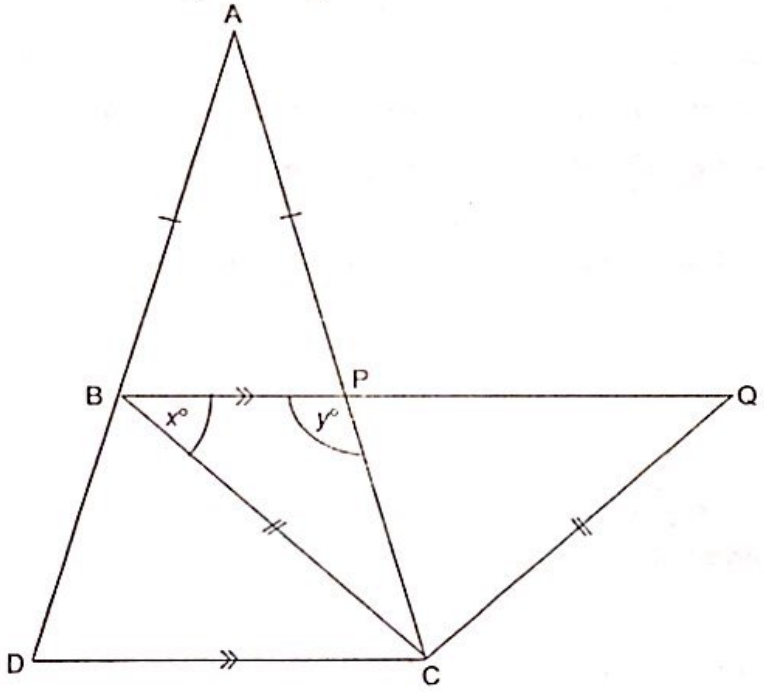
b When I ask them, sixty percent of my Year 9 Mathematics set say they are playing cricket as their summer term sport. Would it be sensible to estimate the proportion of the whole of Year 9 who are playing cricket as their summer term sport as 60%? Explain your answer.

7



In the diagram, the triangle ABC has been cut up into four smaller triangles. Three of these triangles, AYZ, CXY and BXZ, are isosceles, as shown. Find the length of AZ.

8 In the diagram line BQ is parallel to line DC, length AB = length AP and length BC = length CQ. Angle CBQ = $x°$ and angle CPB = $y°$.



a Which other two angles in the figure are equal to $x°$? Justify your answers.

b Find, in terms of x and y, angles
   i BDC
   ii PCQ
   Give a clear reason for each step in your derivations.

c Prove that angle DBC = angle PCQ
   Justify each step in your argument.

9 Tom takes 2 hours and 15 minutes to paint 12 metres of fencing. Ben takes 3 hours and 45 minutes to paint 25 metres of fencing. If the two boys worked together, how long would they take to paint 42 metres of fencing?

10 In a football league, each of twelve teams plays each of the other teams twice: once at home and once away.

a How many games are played altogether?

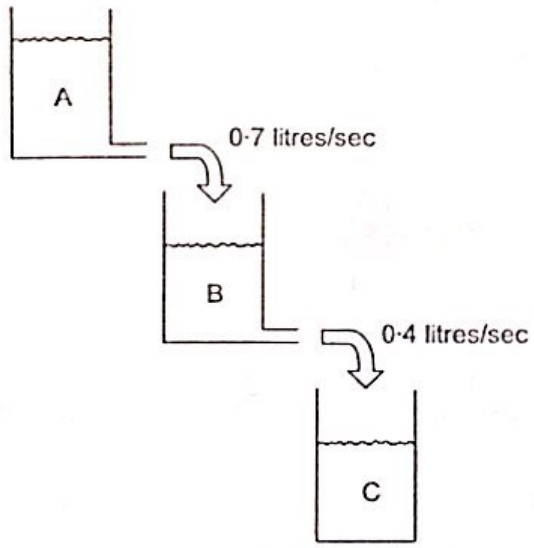
A team gets four points for winning away, three points for winning at home, one point for drawing and no points for losing.

b After one team has played 6 games, they have scored 12 points in total.
There are six possible ways they could have achieved this total score. Copy and complete the table below to show all of them. One possibility has been entered for you.

home wins	away wins	draws	losses	played	points
4	0	0	2	6	12
				6	12
				6	12
				6	12
				6	12
				6	12

c After all the games have been played, the total number of points scored by all the teams is 446. If there were a total of 72 away wins, how many draws were there?

11



The diagram shows how water flows between three vessels. Vessel A empties into vessel B at a constant rate of 0·7 litres per second until vessel A is empty. Vessel B empties into vessel C at a constant rate of 0·4 litres per ~~minute~~ until vessel B is empty.
Initially, vessel A contains 42 litres of water, vessel B contains 10 litres of water and vessel C is empty.
You may assume that none of the vessels overflows at any time.

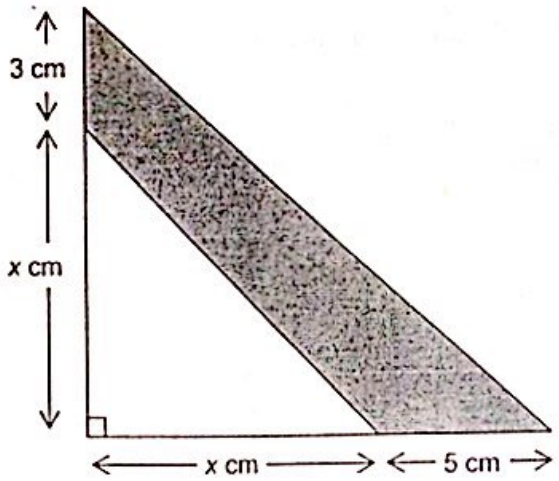
a After how many seconds will vessel A be empty?

b What is the maximum amount of water contained in vessel B while the water is flowing?

c After how many seconds will vessel B be empty?

* SECOND (NOT MINUTE)

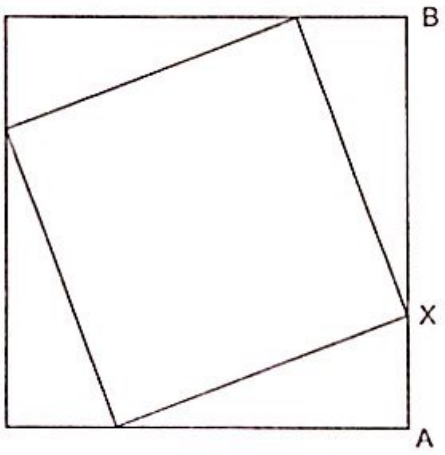
12 The diagram shows a right angled triangle (not to scale).



a Write and simplify an expression in terms of x for the area of the shaded region.

b If the area of the shaded region is $41\frac{1}{2}$ cm^2, find the value of x.

13 The diagram shows two squares.



a If the area of the smaller square is 73 cm^2, and the length AX = 3 cm, find the area of the larger square.

b If the area of the smaller square is 60 cm^2, and lengths AX and XB are in the ratio 1 : 2, find the area of the larger square.