



WESTMINSTER SCHOOL

2011 CHALLENGE

**MATHEMATICS III**

Wednesday 4<sup>th</sup> May 2011

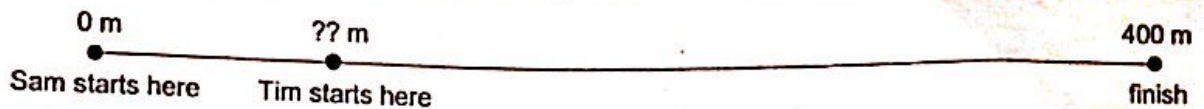
Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

- 1 Sam and his younger brother Tim are having a race on a 400 metre track. Sam gives Tim a head start.



Sam runs at an average speed of  $7.5 \text{ ms}^{-1}$  and Tim runs at an average speed of  $6 \text{ ms}^{-1}$ .  
The two boys start at the same time and they reach the finish at the same time.  
What length of head start did Sam give Tim?

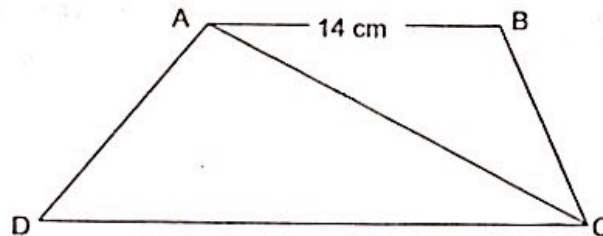
- 2 Solve the simultaneous equations

$$y = x + 4$$

$$3y = 5x + 17$$

- 3 At a wedding ceremony, two fifths of the guests were male.  
A quarter of all the guests at the ceremony were invited to the reception.  
Three eighths of the male guests at the ceremony were invited to the reception.  
a What fraction of the female guests were invited to the reception?  
One hundred and sixteen female guests were invited to the reception.  
b How many male guests were invited to the reception?

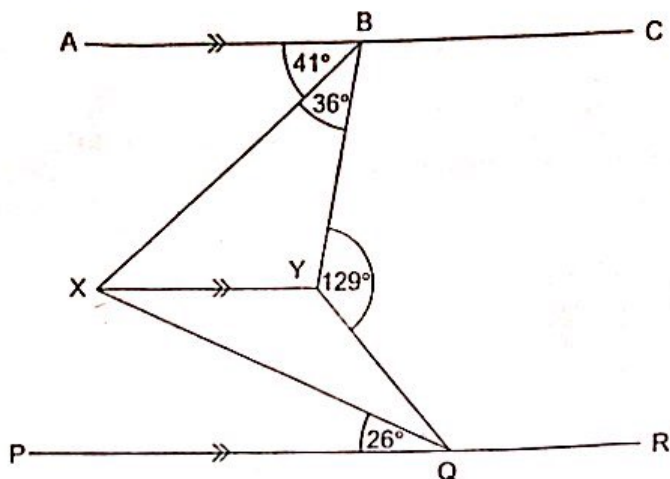
4



In the trapezium ABCD shown above, the length AB is 14 cm.  
The area of triangle ABC is  $31.5 \text{ cm}^2$ , and the area of the trapezium ABCD is  $81 \text{ cm}^2$ .  
Find length CD.

- 5 There are 96 boys in Ben's year at school.  
Ben asks eighteen boys randomly chosen from his year at school whether they prefer donuts or muffins. Twelve of the boys he asks prefer donuts to muffins.  
a Estimate how many boys in Ben's year prefer donuts to muffins.  
Ben asks eight of his friends whether they prefer playing football or listening to music. Seven of these friends prefer playing football to listening to music.  
b Estimate how many boys in Ben's year prefer playing football to listening to music.  
c Which of the estimates in a and b do you think is more reliable? Give two reasons for your answer.
- 6 In my spare time, I drive combine harvesters. My smaller combine harvester has width 2.8 metres and it takes me seven and a half hours to harvest the grain from a 42 acre field with this harvester.  
a How long will it take me to harvest the grain from a 63 acre field?  
I also own a larger combine harvester, with a width of 3.5 metres.  
b If I drive the larger combine harvester at the same speed as the smaller, how long will it take to harvest the grain from a 42 acre field with this harvester? What are you assuming in your calculation?  
If I drive my smaller combine harvester  $X\%$  faster, it takes six and a quarter hours to harvest the grain from a 42 acre field.  
c Find the value of X.

- 7 In the diagram, lines ABC, XY and PQR are all parallel.



- a Find angle XYQ. Give a clear reason for each step in your derivation.  
 b Prove that length XY = length YQ. Justify each step in your argument.
- 8 Consecutive numbers are whole numbers which are next to each other in order, e.g. 11, 12, 13. It is possible to write 15 as a sum of consecutive numbers in three ways:  
 $7+8$   
 $4+5+6$   
 $1+2+3+4+5$   
 Write 105 as a sum of consecutive numbers in as many ways as possible.
- 9 Each square in the grid below is to be filled with a whole number from 1 to 9. The same number cannot appear twice in the same row or twice in the same column. The totals of some of the rows and columns are given. One of the numbers is entered for you.

				Row totals
			c	
				11
	a			7
6	b			30
Column totals	6	13	19	15

- a What is the only possible set of three numbers that can be entered in the third row?  
 b Which two numbers must appear in cells a and b? Explain your answer clearly.  
 c Use the row and column totals to work out what number must appear in cell c.  
 d Show that there is only one way to complete the remaining cells. Explain your reasoning.

**TURN OVER**

- 10
- Factorise  $x^3 - x^2$ .
  - Calculate  $11^3 - 11^2$ .
  - Find the prime factorisation of 21168.
  - Find the whole number  $x$  which satisfies the equation  $x^3 - x^2 = 21168$ .
  - Given that  $n$  is a whole number and that  $21168n$  is an exact square number, find the smallest possible value of  $n$ .

11 The following are the instructions for finding a *Pythagorean triple*.

- Choose two positive whole numbers  $h$  and  $k$ .
- Calculate

$$a = h^2 - k^2$$

$$b = 2hk$$

$$c = h^2 + k^2$$

Then  $a$ ,  $b$  and  $c$  are a Pythagorean triple.

- What are the values of  $a$ ,  $b$  and  $c$  if  $h = 13$  and  $k = 6$ ?
- What are the values of  $h$  and  $k$  if  $a = 95$  and  $b = 168$ ?
- What are the values of  $h$  and  $k$  if  $a = 779$  and  $c = 1021$ ?

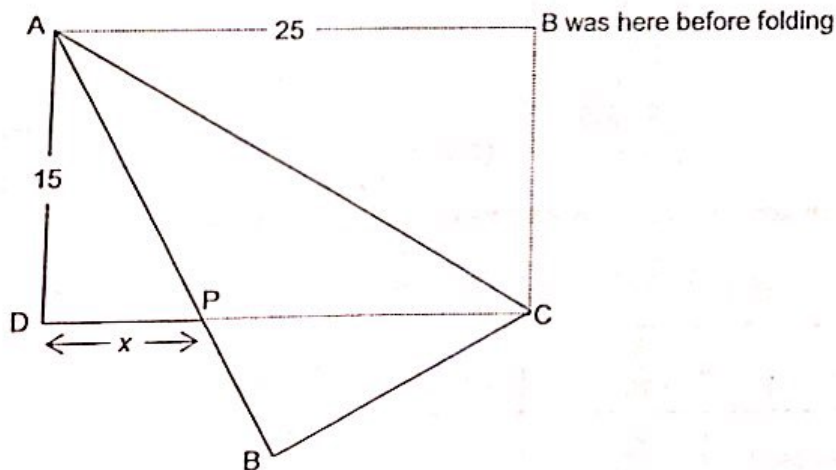
12 I have a large field of grass on my farm, which is sufficient to feed a cow for 14 days and a sheep for 35 days. How long would the field last if I put the cow and the sheep to graze in it?

13 There are a total of 183 boys in a Prep school. Some boys in the school own an iPhone and some own an iPod. Some boys own both an iPhone and an iPod and some own neither an iPhone nor an iPod. You are given that:

- the number of boys who own an iPhone is equal to the number of boys who own an iPod;
- the number of boys who own neither an iPhone nor an iPod is 11 more than the number of boys who own both an iPhone and an iPod.

How many boys own an iPhone?

14 The diagram shows a piece of paper 25 cm by 15 cm, with vertices A, B, C and D, which has been folded along a diagonal.



- Explain why  $(25 - x)^2 = 15^2 + x^2$
- Solve this equation to find the value of  $x$ .