WESTMINSTER SCHOOL

THE CHALLENGE 2012

MATHEMATICS II

Tuesday 1st May 2012

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

1 A beaker is full of glycerin. The beaker and the glycerin together weigh 1030 grams. When one third of the glycerin is poured away, the beaker and the remaining glycerin weigh 715 grams. The density of glycerin is 1·26 grams per cm^3. What is the volume of the beaker?

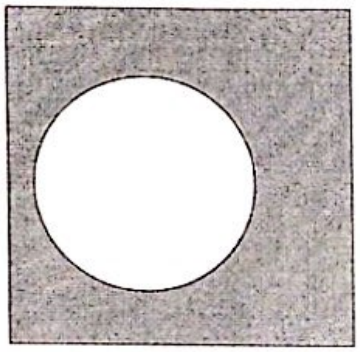
2 a What is two-thirds of $\frac{3x}{4}$?

b Subtract $x - 7$ from $2x - 3$, and simplify your answer.

c Make a the subject of the formula $\frac{v}{a} = t$.

d Solve the equation $2x + 1 = \frac{x}{3}$.

3 The diagram shows a square with side length 7·6 cm.
A circle has been cut out of the square, leaving the shaded region, which has area of 43·9 cm^2.
What is the radius of the circle?



4 a Arthur is 13 years old. His allowance now is 28% greater than it was when he was 11. When he was 12, his allowance was 16% greater than when he was 11. When he was 12, his allowance was £8·70 a week. What is it now?

b Bijan is 16 years old. His allowance when he was 14 was £20·80 a week and his allowance now is £26·10 a week. His allowance went up by the same percentage between the ages of 14 and 15, as between the ages of 15 and 16. What was his allowance when he was 15?

5 A small research submarine with a scientist on board descends from the surface of the Pacific to the bottom of the Mariana trench, which lies 35800 feet below the surface. It leaves the surface at 10:47 and descends at a rate of 234 feet per minute. On seven occasions on the way down the submarine stops so the scientist can conduct observations, stopping for 28 minutes on each occasion.

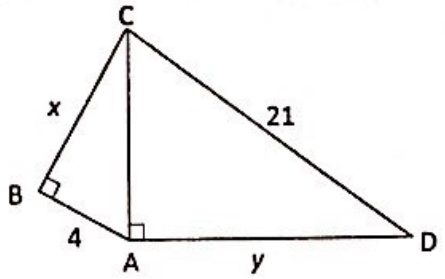
a At what time will the submarine reach the bottom of the trench?

At the bottom of the trench, the scientist conducts observations for a longer period of time and the submarine then ascends, without stopping, at a rate of 351 feet per minute, eventually reaching the surface at 22·03.

b For how long did the scientist conduct observations at the bottom of the trench?

6 When Harry's school organised a trip recently, there were x small coaches, with 32 pupils in each, and y minibuses, with 12 pupils in each. Each coach carried three teachers and each minibus carried two teachers. Altogether, 244 pupils and 29 teachers went on the trip.
Write two simultaneous equations to express this information, and solve them to find the values of x and y.

7 **In the diagram, which is not drawn to scale,** AB = 4 cm, BC = x cm, CD = 21 cm and AD = y cm. ABC and ACD are right angles.



a Show that $x^2 + y^2$ must equal 425.

b Use your calculator to find all possible pairs of positive whole number values of x and y with $x^2 + y^2 = 425$.

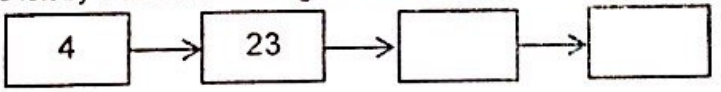
8 Four boys – Arnav, Ben, Conor and Darius – share 44 cookies between them.
Ben eats 4 more cookies than Arnav.
Darius eats three times as many cookies as Conor.
Arnav and Ben between them eat 12 fewer cookies than Conor and Darius do between them.
How many cookies does each boy eat?

9 It takes Paul 110 minutes, with his large polishing machine, to polish the floor of the assembly hall, which is a rectangle 33 metres by 12 metres.

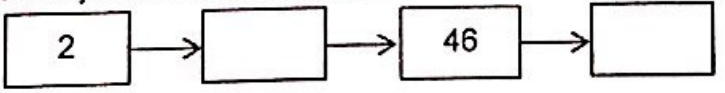
a How long will it take him, with the large polishing machine, to polish the floor of the gym, which is a rectangle 14 metres by 9 metres?

b One day, when Paul is polishing the floor of the assembly hall, his large polishing machine breaks down after 44 minutes. He finishes the job with his small polishing machine, with which it takes him 175 minutes to polish the *whole* floor of the assembly hall. How long does it take Paul to finish the job?

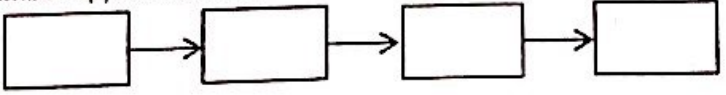
10 **a** The number in each box in the sequence below is found by multiplying the number in the box to the left by 5 and then adding 3. What number should be in the fourth box?



b The number in each box in the sequence below is found by multiplying the number in the box to the left by 3 and then adding a constant amount. What number should be in the fourth box?

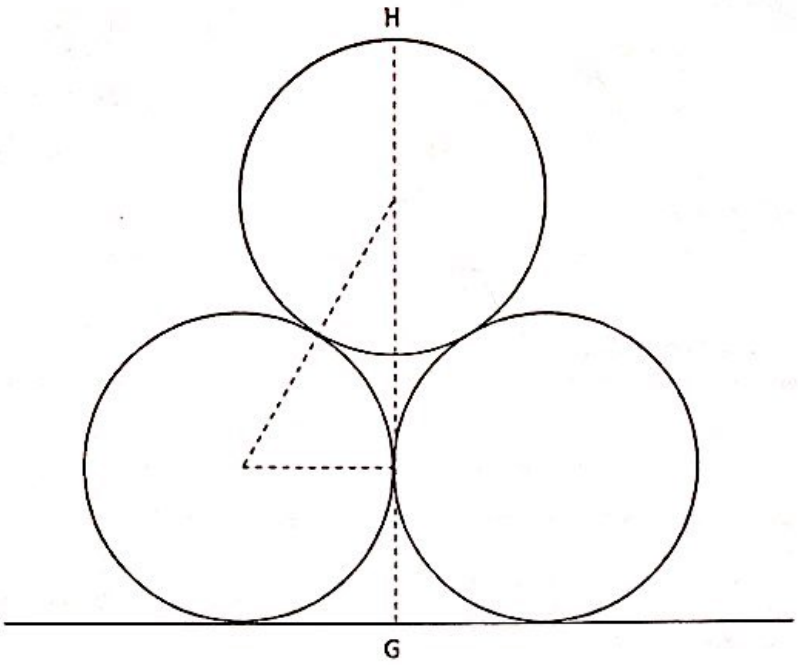


c The number in each box in the sequence below is found by multiplying the number in the box to the left by 4 and then adding 6. What number should be entered in the first box so that the ***same*** number appears in ***all*** the boxes?



TURN OVER FOR QUESTION 11.

11 The diagram shows a side view of three identical cylindrical barrels piled on the ground.



a If the radius of each cylinder is 27·6 cm, find the height GH of the pile.
The dotted lines are drawn to help you decide how to do the calculation.

b If the height GH is 131 cm, find the radius of each cylinder.

WESTMINSTER SCHOOL

THE CHALLENGE 2012

MATHEMATICS III

Wednesday 2nd May 2012

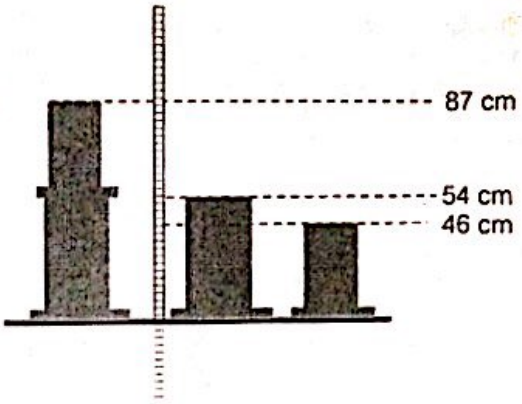
Time allowed: 1 hour 30 minutes

You may not use a calculator in this paper.

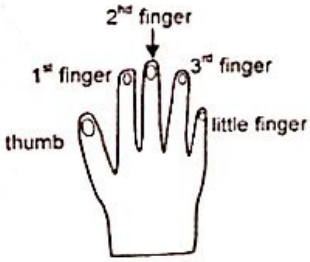
All your working should be clearly shown.

You should attempt all the questions.

1 The diagram shows a metre ruler partly buried in the ground, so that ground level is not at zero height.
Two top hats are measured. The large hat comes up to the 54 cm mark on the ruler and the small hat to the 46 cm mark. When one is placed on the other, they come up to the 87 cm mark.
How tall are the two top hats?



2 A boy counts on his fingers, backward and forwards across his right hand as follows: thumb, 1st finger, 2nd finger, 3rd finger, little finger, 3rd finger, 2nd finger, 1st finger, thumb, 1st finger, ... and so on.



If he starts counting at one, on his thumb, which finger will he be on when he reaches two thousand and twelve?
Explain clearly how you decided.

3 Solve the equation

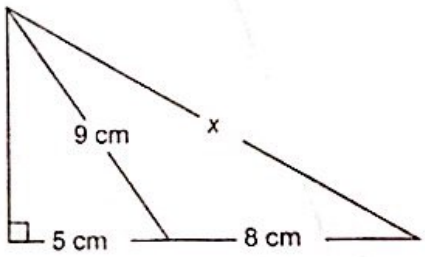
$$\frac{x-2}{3}+\frac{1}{4}=\frac{x}{6}.$$

4 **a** What percentage reduction in price is equivalent to offering "four for the price of three"?

b My horse trots 80% faster than he walks. When he canters he is 25% faster than when he trots but 55% slower than when he gallops.
What is his walking speed as a percentage of his galloping speed?

c When I asked them, five twelfths of my class claimed to have spent more than an hour on their prep. In fact, only three eighths of the class had spent more than an hour on their prep. What percentage of those who claimed to have spent more than an hour on their prep were exaggerating?

5 Find length x in the diagram.



6 In Toby's school, boys are given bonus points for good work in Mathematics.
One term, the mean average number of bonus points achieved by the boys in Toby's year is 25.
There are 120 boys in Toby's year.
For each of the following statements, say whether: you can be sure it is true; it might or might not be true; it definitely isn't true. Explain in each case how you decided.

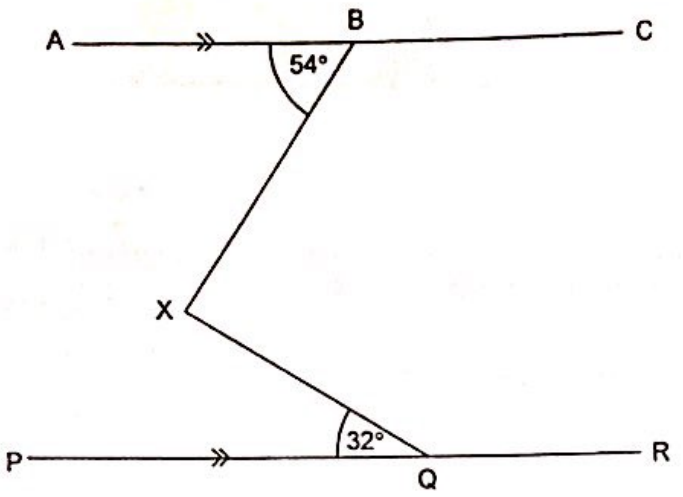
a Most boys in the year gained between 20 and 30 bonus points.

b The same number of boys gained 26 or more bonus points as gained 24 or fewer bonus points.

c Forty boys in Toby's year gained more than 75 bonus points.

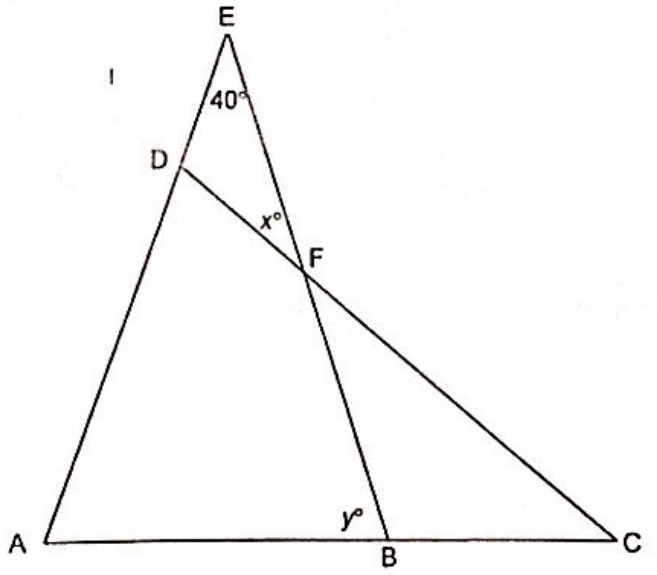
7 The Grand Old Duke of York marched at 4 kilometres per hour from his barracks up to the top of a hill. Then he immediately turned round and marched back again at 6 kilometres per hour. The whole exercise took him three hours and forty-five minutes. How far was it from his barracks to the top of the hill?

8 a In this diagram, lines ABC and PQR are parallel.



i Copy the diagram onto your answer sheet.
ii Find angle BXQ. Give a clear justification for your derivation and show on your copy of the diagram any additional lines you need to draw.

b In the diagram below, angle DEF = 40°, angle EFD = $x°$ and angle ABF = $y°$.



i Find angles CDA and DAC in terms of x and y.
ii Hence show that if $x + y = 100$, then triangle CDA is isosceles.

9 a Louis and his younger brother Matthew are having a race. They both start at the same time, but Matthew starts 100 metres ahead of Louis on the track. Louis runs 4 metres per second faster than Matthew. How long after the start of the race does Louis catch Matthew up?

b Nick and his younger brother Oscar are having a race. They both start at the same place, but Oscar starts 10 seconds before Nick. Nick runs one and a half times as fast as Oscar. How long does it take Nick to catch Oscar up?

10 **a** A prep school is suffering from a flu epidemic. On Monday, 36% of the school has flu. By Tuesday, 11 pupils have recovered from the flu, but another 32 pupils have caught the infection. On Tuesday, 48% of the school has flu. How many pupils are there in the school?

b In a rather insanitary prep school, a large number of pupils suffer from gangrene or botulism (but, fortunately, not both).

On Monday, 25% of those in the infirmary have gangrene and the rest have botulism.
Between Monday and Tuesday, two of the pupils with gangrene recover, but five more pupils with gangrene are admitted, while seven of the pupils with botulism recover and six more pupils with botulism are admitted.
On Tuesday, 30% of the pupils in the infirmary have gangrene and the rest have botulism.

How many pupils were in the infirmary on Monday?

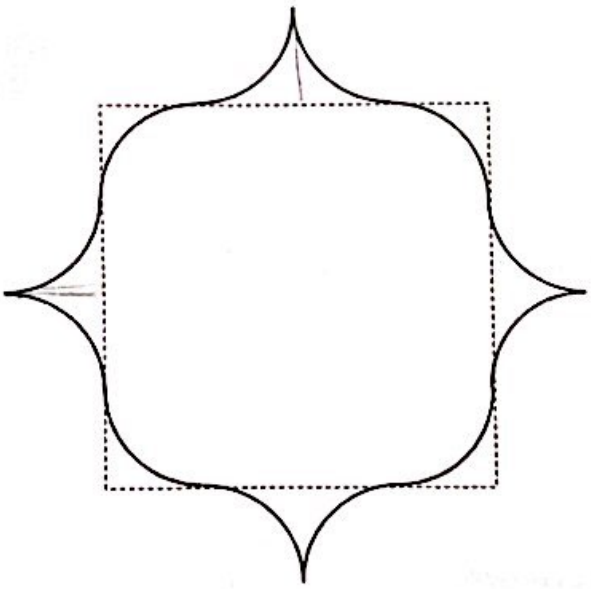
11 **a** Show all the different ways that the digits from 1 to 9 can be divided into three groups of three so that one group adds up to 8, one group adds up to 18 and one group adds up to 19.

Set out your answers as shown below. One possibility has been entered for you.

	three digits which add up to		
	8	18	19
1st possibility	1, 2, 5	3, 6, 9	4, 7, 8
2nd possibility			
etc			

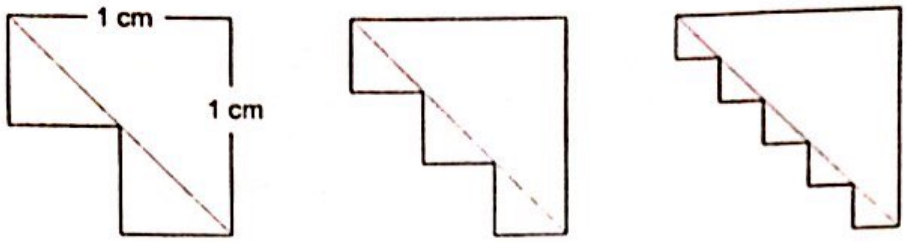
b Find three three-digit numbers which between them contain all the digits from 1 to 9 once each, and which add up to 999. Explain clearly how your answer could be found using part **a** of this question.

12 The diagram shows a shape whose perimeter consists of twelve quarter-circles, all from circles of radius 1 centimetre.
What is the area of the shape? Leave π in your answer.



TURN OVER

13 Each of the three diagrams below shows an isosceles right angled triangle with "teeth" added to the hypotenuse. The first has two "teeth" on the hypotenuse; the second, three "teeth" and the third, five "teeth". In each case, both the shorter sides of the original right angled triangle are 1 cm long, and the "teeth" are equal-sized isosceles right angled triangles.



a Find the area and the perimeter of each of the three shapes.
b What would be the area and perimeter of a shape with n teeth?

14 **a** Multiply out $(3x+1)(2x+1)$.
b By substituting $x = 10$ in the result from part a, find two two-digit numbers whose product is 651.
c Multiply out $(3x+1)(2x+3)$.
d What three digit number does the result from part c enable you to factorise in the same way?
e Find two brackets you could multiply out that would enable you to factorise 527 in the same way.