Eton College King's Scholarship 2003 B solutions

1) a) i) $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{7}{10}, \frac{5}{6}$

ii) $2\sqrt{10},\ 3\sqrt{5}, 4\sqrt{3}, 7, 5\sqrt{2}$

b) 2^5 and 5^5 (32 and 3125)

2) a) 280

b) i) $4x + 4x = 56$

$x + y = 14$

$x^2 - y^2 = 56$

ii) $\frac{x^2-y^2}{x+y} = x - y = \frac{56}{14} = 4$

$x = 9, y = 5$

3) 40

4) a) 5

b) i) 7

ii) 3

5) a) Multiply top and bottom of the expression for x_2 by x_1

b) $x_3 = \frac{x_1}{1+2\,x_1}$, $x_4 = \frac{x_1}{1+3\,x_1}$

c) $x_{10} = \frac{2}{19}$

6) a) $(x-y)^2 \geq 0$ so $x^2 - 2xy + y^2 \geq 0$ hence result

b) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + xz + yz)$

$\leq x^2 + y^2 + z^2 + (x^2 + y^2) + (x^2 + z^2) + (y^2 + z^2)$

$= 3(x^2 + y^2 + z^2)$

7) a) $\frac{1}{3-v} + \frac{1}{3+v}, \frac{1}{4-2v} + \frac{1}{4+2v}$

b) $v^2 = 1.5$

c) 48 min

8) a) i) $\frac{a+b}{ab}$

ii) $a^2 + 2ab + b^2$

b) i) 2

ii) 90

iii) 850

9) a) 6

b) 5

c) 2

d) $\frac{4}{5}$

e) $\frac{12}{25}$

10)a) 10

b) 5+2n

c) $n^2 + (5 + 2n)^2 = 50$

$n^2 + 4n - 5 = 0$

d) $n = 1$