Eton B 2004 (sketch solutions only) Tom Linton

1) a)

	2^1	2^2	2^3	2^4	2^5
end digit	2	4	8	6	2, ...

Every 4th power of 2 that is a multiple of 4 ends in 6.

2^{2004} ends in 6.

b) 3: 3, 9, 7, 1, 3, ... (1)

4: 4, 6, 4, ... (6)

5: 5, 5, ... (5)

6: 6, 6, ... (6)

c) $1^{2004} + \ldots + 6^{2004}$ ends in: $1+6+1+6+5+6 = 25$ → ends in 5 so divisible by 5.

2) a) $\sqrt{2x^2+2y^2}$ b) x^2+y^2 c) 10 cm^2

3) a) $(a-b)^2+(a-c)^2+(b-c)^2 \geq 0$

$a^2+b^2-2ab+a^2+c^2-2ac+b^2+c^2-2bc \geq 0$.

$a^2+b^2+c^2 \geq ab+ac+bc$

b) $(a^2+b^2+c^2)(a+b+c) \geq (ab+ac+bc)(a+b+c)$

$a^3+a^2b+a^2c+ab^2+b^3+b^2c+ac^2+abc+bc^2+abc+a^2b+ab^2+abc+a^2c+abc+ac^2+abc+b^2c+bc^2$

$a^3+b^3+c^3 \geq 3abc$ if $a+b+c \geq 0$

4) a) $(n+3)(p-15) = 1080$

b) $p = S_n$

c) $n = 15,\ p = 75$

5) a) $s_1 = \frac{d}{6}$, $s_2 = \frac{d}{10}$ (mph) b) d

c) $s_1 t + s_2 t = d$ $\quad \frac{dt}{6} + \frac{dt}{10} = d$ $\quad t = 3h\ 45m \Rightarrow 15{:}45$

6) a) $\angle XZ = 90°$ $\quad XB = \sqrt{3}$ cm b) 3cm c) Pythag $\Rightarrow r^2 + h^2 = 3$

d) $r = 2, h = 1$ e) $3:1$

7) 10cm, 6cm $\left[(x+4) - \frac{3\frac{1}{2}}{5}(x+4) = \frac{1}{2}x\right]$

8) a) Circle b) 90°, 2m c) 1.44π m

9) a) Each letter can appear in combination with all four others
$\Rightarrow$ answer. Or write out combinations.

b) Nine totals add to £395

$395 \div 4$ has remainder 3. Therefore $(a+b) \div 4$ has remainder 1.

Only 37 and 61 work. $a+b = 61 \Rightarrow 4\left(\underbrace{a+b}_{=61} + \underbrace{c+d}_{=61} + e\right) =$
$488 + 4e$ which is too big.

c) $e = 34$

d) e Largest $61 = 34 + \underline{27}$

$a+b = c+d = 37$ so $27 + \underline{10} = 37$

$10 + \underline{13} = 23$

$13 + \underline{24} = 37$

10, 13, 24, 27.

10] a) $(ab+ac+bc)^2 = a^2b^2 + a^2c^2 + b^2c^2 + 2(a^2bc + ab^2c + abc^2)$

So $a^2b^2 + a^2c^2 + b^2c^2 = (ab+ac+bc)^2 - 2abc(a+b+c)$

b) $V = abc = 6$

$SA = 2(ab+ac+bc) = 22 \Rightarrow$ (i) $ab+ac+bc = 11$

Total of side lengths $= 4(a+b+c) = 24 \Rightarrow a+b+c = 6.$

ii) New $SA = 2(a^2b^2 + a^2c^2 + b^2c^2)$

[from (a)] $= 2[(ab+ac+bc)^2 - 2abc(a+b+c)]$

$= 2[(11^2) - 2 \times 6 \times 6]$

$= 98 \text{ cm}^2$