Eton College King's Scholarship 2008 B solutions

1) a) 40 degrees
   b) ACF = 80, CFA = 40, FAC = 60

2) a) x = 5, y = -11
   b) $p = \pm\sqrt{5},\ q = \frac{1}{11}$

3)

	3 faces	2 faces	1 face	0 faces	Total
a)	8	24	24	8	64
b)	8	$12(x-2)$	$6(x-2)^2$	$(x-2)^3$	x^3
c)	24	$12(x-4)$	$6(x-2)^2$	$(x-2)^3$	x^3-8

4) a) ABX = x (isosceles)
   AXB = 180-2x (angles in triangle are 180)
   BXC = 2x (angles on straight line are 180)
   BCX = 2x (isosceles)
   b) Let BAX = x
   ABX = x (isosceles)
   AXB = 180-2x (angles in triangle are 180)
   BXC = 2x (angles on straight line are 180)
   XBC = 90-x (XBC isosceles and angles add to 180 in triangle)
   ABC = ABX + XBC = x + (90-x) = 90

5) a) x = 6
   b) 28 years old

6) a) Let the distance between E and A be d.
   $Whole\ time\ taken = \frac{d}{50} + \frac{d}{75} = \frac{d}{30}$
   $Average\ speed = \frac{2d}{\frac{d}{30}} = 60km/h$

   b) $\frac{d}{90} = t$
   $\frac{d}{70} = t + \frac{1}{15}$
   $\frac{d}{315} = \frac{1}{15}$
   $d = 21km$

7) a) $x^2 + y^2 = (y+1)^2$
   $x^2 + y^2 = y^2 + 2y + 1$
   $2y = x^2 - 1$
   $y = \frac{x^2-1}{2}$
   b) For y to be whole $x^2 - 1$ must be a multiple of 2.
   So x^2 must be odd.
   So x must be odd.

c) 3,4,5
5,12,13
7,24,25
9,40,41
11,60,61
13,84,85
15,112,113

8) a) $\frac{1}{m}+\frac{1}{n}=\frac{1}{6}$
$6n+6m=mn$
$mn-6m-6n=0$
$mn-6m-6n+36=36$
$(m-6)(n-6)=36$

b)

m-6	n-6	m	n
36	1	42	7
18	2	24	8
12	3	18	9
9	4	15	10
6	6	12	12

9) a) $x^2+y^2=89$
b) 64

10)a) $h=\sqrt{2}$
b) $8\sqrt{2}$