

# Eton College King's Scholarship Examination 2011

(One and a half hours)

## MATHEMATICS A

*Answer Question 1 and as many of the other five questions as you can.*

*Question 1 is worth 50 marks. All other questions are worth 10 marks each.*

*Show all of your working. The use of calculators is permitted.*

1. **Compulsory Question**

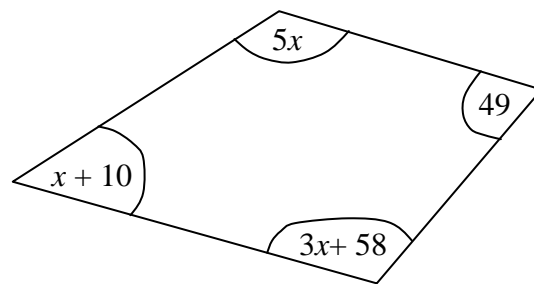
(a) The mean of 3 numbers is 15 and the gap between the middle and the largest number is twice the gap between the middle and the lowest number. If the largest number is 25, find the lowest number. [3]

(b) If  $p = 5$  and  $q = 4$ , evaluate the following, leaving your answers as exact fractions:

(i)  $\frac{2}{p} + \frac{5}{q}$  [2]

(ii)  $\frac{2(p-q)}{4p+5q}$  [2]

(c) The diagram below shows a quadrilateral. Calculate the value of  $x$ . [3]



(d) (i) Simplify the expression  $\frac{2x+12}{2} - x$  [2]

(ii) Evie asks me to think of a number, multiply it by 2, then add 12, then divide by 2 and then subtract the number with which I started.

When I do this, I get the answer 7. Use your answer to (i) to explain why I must be wrong. [2]

(e) Solve the following equations:

(i)  $\frac{5-3x}{4} = 17$  [2]

(ii)  $\frac{5x+11}{2} - \frac{1}{3}(x+4) = 8$  [3]

(f) Solve the simultaneous equations

$$5x - 11y = 47$$

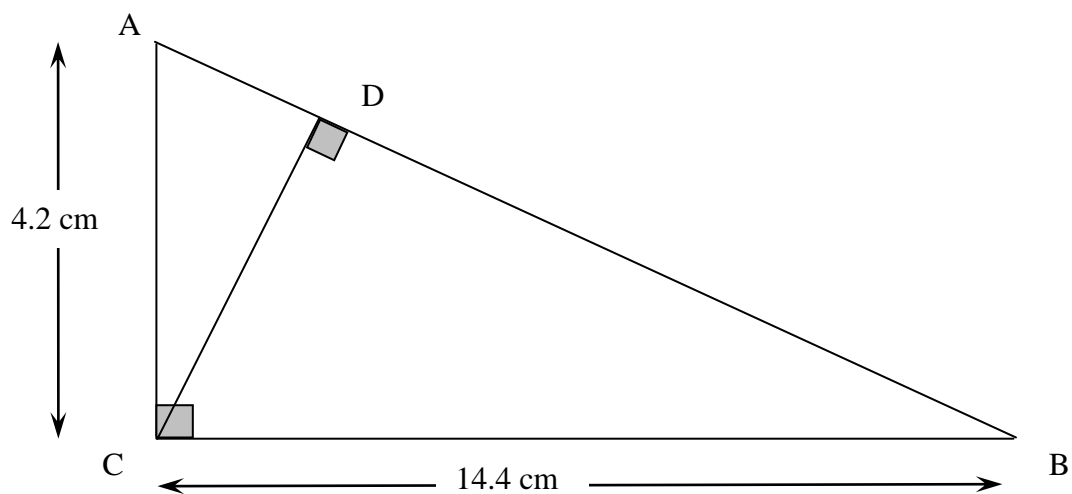
$$3x + 7y = 1$$

(g) Solve the following inequalities:

(i)  $7x - 2 < 19$  [2]

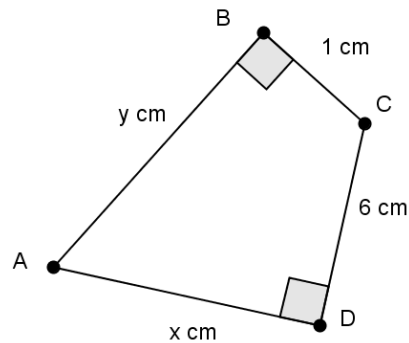
(ii)  $x - 1 < \frac{1}{3}x$  [2]

- (h) A new savings account offers 3.2% interest for the first year followed by 6.5% interest rate for the second year. Investors may only put money into the account at the start of the two years and may not withdraw money from the account until the end of the second year. The interest after one year is simply added to the account at the end of the first year.
- If Evie invests £30000, what interest will be added to Evie's account at the end of the first year? [2]
  - If £1440 interest is added to Iris's account after the first year, how much did she invest originally? [2]
  - If Rachel receives £16770 interest at the end of the second year, how much interest was added to her account at the end of the first year? [3]
- (i) The diagram shows a right-angled triangle ABC. The point D lies on side AB and the angles ACB and CBD are both right angles.



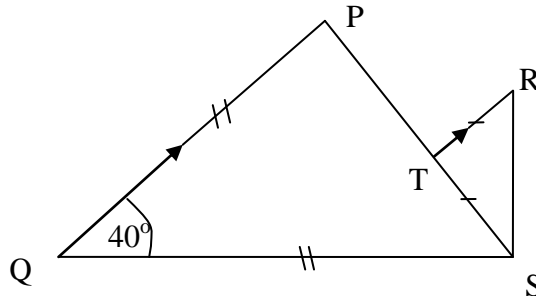
- Find the length AB. [2]
  - Find the area of the triangle ABC. [2]
  - Find the length CD. [2]
- (j) Given the formula  $R = ap^2 - q^2$ ,
- find  $R$  when  $p = 3$ ,  $q = -5$  and  $a = 2$ ; [2]
  - make  $a$  the subject of the formula; [2]
  - show that if  $R = q^2$  and  $p = 2q$ , show that  $a = \frac{1}{2}$ . [2]
- (k) (i) What do you multiply  $4\frac{2}{3}$  by to get  $1\frac{3}{5}$ . Give your answer as an exact fraction. [2]
- (ii) What do you multiply  $x^2$  by to get  $\frac{2}{3}x$ ? [2]

2. (a) Expand the brackets and simplify:  $(y - x)(y + x)$ . [1]
- (b) If  $a$  and  $b$  are positive whole numbers, find all pairs of values of  $a$  and  $b$  with  $a > b$  that solve the equation:  $ab = 35$ . [2]
- (c) In the quadrilateral shown, the angles  $ABC$  and  $CDA$  are both right angles. The side  $BC$  has length 1 cm and the side  $CD$  has length 6 cm. Let  $AB = y$  cm and  $AD = x$  cm where  $x$  and  $y$  are WHOLE numbers.

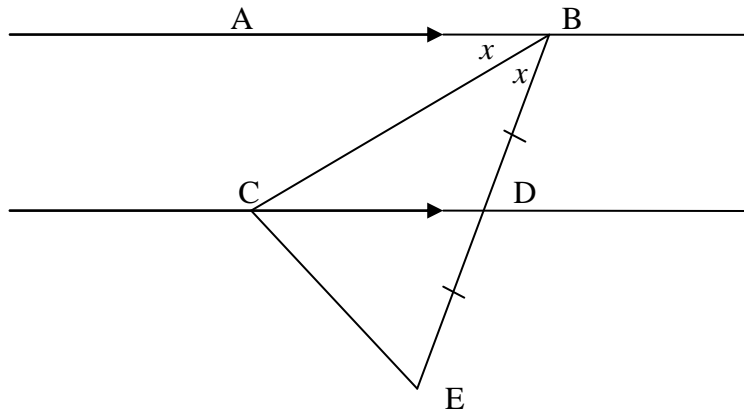


- (i) Use Pythagoras' Theorem to show that  $y^2 - x^2 = 35$ . [2]
- (ii) Use your answers to parts (a) and (b) to find all possible values of  $y + x$ . [2]
- (iii) By finding the corresponding values of  $y - x$  and using simultaneous equations, find all possible pairs of values for the length of sides  $AB$  and  $AD$ . [3]
3. David decides to start listing the digits of the numbers from 0 to 2011 in order so that his list starts  
 0123456789101112131415161718192021222324.....  
 Notice that the 17th digit that he writes is a 1.
- (a) Making your reasoning clear, show that the 50th digit he writes is a 9. [2]
- (b) (i) How many 2 digit numbers does he write? [1]
- (ii) Show that by the time he has finished writing the digits of the number 99, he will have written 190 digits. [2]
- (c) Show that by the time he has finished writing the digits of the number 999, he will have written 2890 digits. [2]
- (d) Explain clearly how many digits David writes in total. [3]

4. (a) In the diagram below, the lines PQ and RT are parallel.  
 The lengths PQ and QS are equal and the lengths RT and TS are equal. Angle PQS is  $40^\circ$ .
- (i) Find the angle TRS, giving reasons. [3]  
 (ii) Find angle RST, giving reasons. [2]



- (b) In the diagram below, the lines AB and CD are parallel. D is the midpoint of BE.  
 Given that angle ABC is equal to angle CBD, show that angle BCE is  $90^\circ$ . [5]



5. (a) Write 777 as a product of its prime factors [2]  
 (b) Two numbers, both two digits long and ending with the same digit as each other, multiply to give a three digit number with each digit equal as shown below.

$$\begin{array}{r} a \quad b \\ \times c \quad b \\ \hline d \quad d \quad d \end{array}$$

- (i) Explain why one of the two digit numbers *must* be either 37 or 74. [2]  
 (ii) Explain why 74 is in fact impossible. [3]  
 (iii) Find the correct pair of two digit numbers. [3]

6. Albert, Barney, Charlie, David and Edward all play exactly one game of draughts against each other. A player gains 3 points if he wins a game, zero points if they lose, and 1 point for a draw.
- (a) How many games are played in total? [2]
  - (b) Show that if there were 5 draws, then in total the players would have scored 25 points. [1]
- In fact, after the games have been played, Albert has 1 point, Barney 3 points, Charlie 4 points, David 9 points, and Edward 10 points.
- (c) How many games ended in a draw? [2]
  - (d) Explain why David could never had a draw. [1]
  - (e) How many games ended in a draw:
    - (i) for Albert; [1]
    - (ii) for Edward. [1]
  - (f) Explaining your reasoning, find the result of the match between Charlie and David. [2]

**[END OF PAPER]**