

Eton College King's Scholarship Examination 2019

MATHEMATICS B

(One and a half hours)

Candidate Number:.....

Please answer on the paper in the spaces provided.

Please write your candidate number on every sheet.

Each question is worth 10 marks.

Show all your working.

Answers without sufficient working may receive little or no credit.

The use of calculators is permitted.

Do not turn over until told to do so.

1. Throughout this question, assume that:

716 gallons = 3255 litres

535 miles = 861 kilometres.

a) How many gallons are there in 82 litres? Leave your answer to 1 decimal place.

b) How many miles are there in 1117 kilometres? Leave your answer to 1 decimal place.

A car engine runs on unleaded petrol, consuming $\frac{37}{500}$ litres per mile. $\frac{3}{4}$ of a tank of fuel allows the car to travel 956 kilometres.

c) If unleaded petrol costs £5.81 per gallon, how much does it cost to completely fill the fuel tank from empty, to the nearest penny?

[10 marks]

2. a) On Monday Albert runs 3 km. One week later he runs the same distance, but his average speed is 20% faster. If his total running time is reduced by 2 minutes 24 seconds, what was his original running time (in minutes and seconds)?
- b) Cath runs the same distance on Thursday, Friday and Saturday. On Saturday she runs 15% faster than on Friday and her total running time is y minutes less. On Thursday she runs 15% slower than on Friday and her total running time is z minutes more. Show, with clear justification, that $23y = nz$ for an integer n which you should calculate.

[10 marks]

3. a) Expand the following, simplifying fully:

(i) $(a - b)^2$

(ii) $a(a - b) - b(a - b)$

(iii) $(a - b)^3$

Consider the following pair of simultaneous equations:

$$\begin{aligned}x^2 - xy &= 238 \\xy - y^2 &= 189\end{aligned}$$

b) By factorising the left-hand side of both equations, show that

$$(x - y)^2 = 49$$

c) Given additionally that x and y are positive and $x > y$, find x and y .

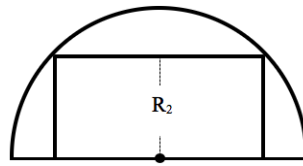
d) Consider the following system of simultaneous equations. By how much do u and v differ?

$$\begin{aligned}u^3 - 3u^2v &= 16 \\v^3 - 3uv^2 &= -11\end{aligned}$$

[10 marks]

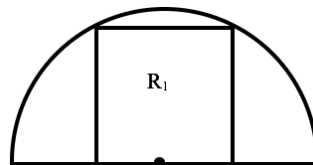
4. a) Figure A shows a semicircle of radius 1 with an inscribed rectangle, R_2 , formed from two adjacent congruent squares. Show the area of R_2 is 1.

Figure A



- b) Figure B shows a semicircle of radius 1 with an inscribed square, R_1 . Find the area of R_1 .

Figure B



- c) Figures C and D also show semicircles of radius 1 with inscribed rectangles R_3 and R_5 , formed from 3 and 5 adjacent congruent squares, respectively. Find the ratio of the areas of $R_3:R_5$, leaving your answer in the form $m:n$ in lowest form for integers m and n .

Figure C

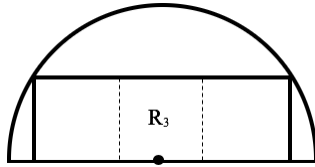
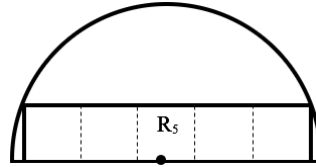


Figure D



[10 marks]

5. When Margaret reached her 17th birthday, Meredith was four times older than Marion.

When Margaret reaches her 25th birthday, eleven times the difference of Meredith and Marion's ages will equal five times the sum of their ages.

- a) Find the ages of Meredith and Marion on Margaret's 17th birthday.
- b) Margaret's current age divides Meredith's age exactly. Given Margaret is younger than 25, determine her current age.

- c) Claire is the daughter of Anne. The product of their ages is 1885. How old was Anne when Claire was born?

[10 marks]

6. Suppose that p and $p + 2$ are both prime numbers.
- a) Is the integer between p and $p + 2$ odd or even? Explain your answer.

b) Expand the following, simplifying fully:

i) $(2n)^2$

ii) $(2n + 1)^2$

For the remainder of this question, assume additionally that the integer between the primes p and $p + 2$ is a perfect square.

- c) Explain why there exists an integer n such that $4n^2 - 1 = p$.

- d) By considering $(2n - 1)(2n + 1)$, find the only possible value of p .

[10 marks]

7. Marcus is making cupcakes for a charity sale. The ingredients cost 15p per cake. Suppose that each cake is sold for $\pounds C$ and that Marcus sells n cakes.
- a) Write down a formula for the total profit $\pounds P$ Marcus makes, in terms of C and n .

90 people will attend the charity sale. If Marcus charges nothing at all for his cakes, 75 people will take a cake (and Marcus will make no money). For each pound above zero Marcus charges per cake, 20 fewer people will buy a cake.

- b) Explain why $n = -20C + 75$
- c) Show that $P = -20C^2 + 78C - 11.25$

- d) By expanding $(C - 1.95)^2$, find a number k such that

$$P = -20(C - 1.95)^2 + k$$

- e) What price should Marcus charge for each cupcake if he wishes to raise the most money possible for charity? Explain your answer.

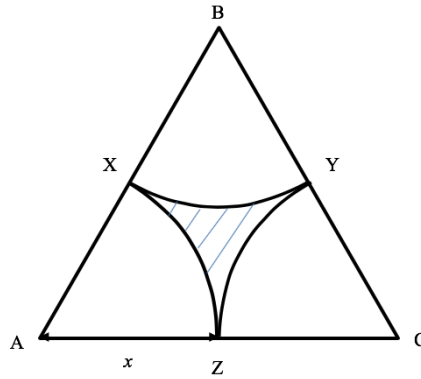
[10 marks]

8. Figure E shows an *Astrum* floor tile. ABC is an equilateral triangle. XZ, XY and YZ are all circular arcs (portions of complete circles) centred at A, B and C respectively.

Lengths $AX = XB = BY = YC = CZ = ZA = x$.

- a) Show the area of triangle ABC equals $\sqrt{3}x^2$.

Figure E

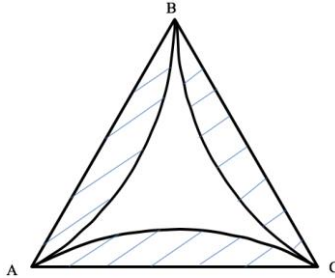


A tessellation of a flat surface (such as a floor) is a tiling that covers the surface using geometric shapes (the tiles) with no overlaps and leaving no gaps.

- b) A large floor is tessellated using *Astrum* tiles. What percentage of the floor will be shaded? Give your answer to 2 decimal places.

- c) Figure F shows a *Folium* floor tile. ABC is an equilateral triangle. Circular arcs AB, BC and CA all have radii equal to the triangle side length AB. If a large floor is tessellated using *Folium* tiles, what percentage of the floor will be unshaded? Give your answer to 2 decimal places.

Figure F



[10 marks]

END OF PAPER