

Sixth Form Entrance 2019

Mathematics

1 hour

Attempt all questions if possible. Do not worry if there are topics you have never covered; do your best on whatever you can attempt.

Questions are not necessarily in order of difficulty.

Marks for parts of questions are given in brackets as a guide.

Show as much working as you can. Calculators are allowed and their use expected.

There is a list of formulae given (last page), not all of which need necessarily be used in this paper.

The paper has twenty-eight questions. Work quickly.

There are one hundred and thirteen marks in total – roughly two a minute.

NAME:

AGE:

PRESENT SCHOOL:

Q1.

Bill's weight decreases from 64.8 kg to 59.3 kg.

Calculate the percentage decrease in Bill's weight. Give your answer correct to 3 significant figures.

.....

(Total for Question is 3 marks)

Q2.

Write the following numbers in order of size. Start with the smallest number.

 0.038×10^2 3800 × 10⁻⁴ 380 0.38×10^{-1}

.....

(Total for Question is 2 marks)

There are yellow discs, red discs, blue discs and green discs in a bag. Dinesh is going to take at random a disc from the bag.

The table shows each of the probabilities that Dinesh will take a red disc, or a blue disc, or a green disc.

Colour	yellow	red	blue	green
Probability		0.40	0.25	0.15

(a) Work out the probability that he will take a yellow disc.

.....

(2)

Dinesh takes at random a disc from the bag. He writes down the colour of the disc. He puts the disc back into the bag.

He will do this 60 times.

(b) Work out an estimate for the number of times he takes a red disc from the bag.

.....

(2)

(Total for Question is 4 marks)

Q3.

The scatter graph shows information about 8 people. It shows each person's height and the circumference of their head.



The table gives this information for 2 other people.

Height (cm)	180	170
Circumference of head (cm)	72	65

On the scatter graph, plot the information from the table. (a)

Describe the correlation. (b)

(1)

(1)

(d) Estimate the circumference of the head of a person who is 156 cm tall.

..... cm (1)

(Total for Question is 4 marks)

Q5.

Julie is x years old. Kevin is x + 3 years old. Omar is 2x years old.

Write an expression, in terms of *x*, for the mean of their ages.

.....

(Total for Question is 2 marks)

Q6.

3x + 5 > 16

x is an integer.

Find the smallest value of x.

.....

(Total for Question is 3 marks)

Q7.

(a) Complete the table of values for $y = \frac{1}{2}x + 4$

x	-2	-1	0	1	2	
У	3		4			
						(2)

(b) On the grid, draw the graph of $y = \frac{1}{2}x + 4$



(2)

(c) (i) On the grid, draw the line that is perpendicular to $y = \frac{1}{2}x + 4$ and passes through the point with coordinates (0, 4). (ii) Find the equation of this line.

Q8.

(a) Expand 4(3x + 5)
(b) Expand and simplify 2(x - 4) + 3(x + 5)
(c) Expand and simplify (x + 4)(x + 6)

(2) (Total for Question is 5 marks)

Q9.

The diagram shows a circle drawn inside a square.



Diagram NOT accurately drawn

The circle has a radius of 6 cm. The square has a side of length 12 cm. Work out the shaded area. Give your answer in terms of π .

.....cm²

(Total for Question is 3 marks)

Q10.

(a) Solve 3(2p-5) = 21

(b) Solve 9x - 11 = 5x + 7

ρ =.....(3)

(Total for Question is 6 marks)

Q11.

A pile of sand has a weight of 60 kg. The sand is put into a small bag, a medium bag and a large bag in the ratio 2 : 3 : 7

Work out the weight of sand in each bag.

small bag kg

medium bag kg

large bag kg

(Total for Question is 3 marks)

Q12.

Make t the subject of the formula

$$p = \frac{3 - 2t}{4 + t}$$

.....

Q13.

$$\frac{(6-\sqrt{5})(6+\sqrt{5})}{\sqrt{31}}$$

Rationalise the denominator of

Give your answer in its simplest form.

.....

(Total for Question is 3 marks)



(a) Reflect shape **P** in the line x = 3



(b) Describe fully the single transformation that maps shape A onto shape B.

.....

(3)

(2)

(Total for Question is 5 marks)



.....

(1)

(Total for Question is 2 marks)

Q16.

There are 200 students at a college. Each student studies one of Art, Graphics or Textiles.

Of the 116 female students, 26 study Graphics. 22 male students study Textiles. A total of 130 students study Art.

The number of students who study Graphics is the same as the number of students who study Textiles.

Work out how many **male** students study Art.

.....

Q17.



A, B and C are 3 service stations on a motorway.

AB = 25 miles BC = 25 miles

Aysha drives along the motorway from A to C.

Aysha drives at an average speed of 50 mph from *A* to *B*. She drives at an average speed of 60 mph from *B* to *C*.

Work out the difference in the time Aysha takes to drive from *A* to *B* and the time Aysha takes to drive from *B* to *C*.

Give your answer in minutes.

.....

(Total for Question is 3 marks)

Q18.

Here are seven tiles.

$$\left(\begin{array}{c}1\\1\end{array}\right)\left(\begin{array}{c}2\\1\end{array}\right)\left(\begin{array}{c}2\\2\end{array}\right)\left(\begin{array}{c}2\\2\end{array}\right)\left(\begin{array}{c}3\\3\end{array}\right)\left(\begin{array}{c}3\\3\end{array}\right)$$

Jim takes at random a tile. He does **not** replace the tile.

Jim then takes at random a second tile.

(a) Calculate the probability that both the tiles Jim takes have the number 1 on them.

.....

(b) Calculate the probability that the number on the second tile Jim takes is greater than the number on the first tile he takes.

.....

(3)

(2)

(Total for Question is 5 marks)

The diagram shows a pyramid.



Diagram NOT accurately drawn

BCDE is a square with sides of length 10 cm.

The other faces of the pyramid are equilateral triangles with sides of length 10 cm.

(a) Calculate the volume of the pyramid.

Give your answer correct to 3 significant figures.

(b) Find the size of angle DAB.

.....

(2)

(4)

(Total for Question is 6 marks)

 $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots cm^3$

Q19.

Q20.



The diagram shows part of a pattern made from tiles.

The pattern is made from two types of tiles, tile A and tile B. Both tile A and tile B are regular polygons. Work out the number of sides tile A has.

Q21.

Here are the first five terms of an arithmetic sequence.

4 11 18 25 32 (a) Write down, in terms of *n*, an expression for the *n*th term of this sequence. An expression for the *n*th term of another sequence is $3n^2 - 1$ (b) Find the fourth term of this sequence.

(2)

(Total for Question is 4 marks)

Q22.

Ella is designing a glass in the shape of a cylinder.

The glass must hold a minimum of $\frac{1}{2}$ litre of liquid.

The glass must have a diameter of 8 cm.

Calculate the minimum height of the glass.



Diagram NOT accurately drawn

Q23.

The diagram shows the triangle PQR.



Diagram NOT accurately drawn

PQ = x cm PR = 2x cmAngle $QPR = 30^{\circ}$

The area of triangle $PQR = A \text{ cm}^2$

Show that $x = \sqrt{2A}$

(Total for Question is 3 marks)



Diagram NOT accurately drawn

M and *N* are two points on the circumference of a circle centre *O*. The straight line *AMB* is the tangent to the circle at *M*.

Angle MON = y

Prove that angle $BMN = \frac{1}{2} y$

(Total for Question is 5 marks)





ABCD is a parallelogram.

DC = 5 cmAngle $ADB = 36^{\circ}$

Calculate the length of AD.

Give your answer correct to 3 significant figures.

.....

Q26.

(a) Solve $2x^2 + 9x - 7 = 0$

Give your solutions correct to 3 significant figures.

(3)

(b) Solve $\frac{2}{y^2} + \frac{9}{y} - 7 = 0$

Give your solutions correct to 3 significant figures.

.....

(2)

(Total for Question is 5 marks)

Q27.

The ratio (y + x): (y - x) is equivalent to k: 1

Show that

$$y = \frac{x(k+1)}{k-1}$$

Q28.

Solve the simultaneous equations

$$x^2 + y^2 = 25$$

y = 2x + 5

End of exam

[Total for paper = 113 marks]

Formula Sheet



Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule:
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of a triangle =
$$\frac{1}{2} ab \sin C$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$



Sixth Form Entrance 2018

MATHEMATICS

1 hour

Write in this booklet.

Attempt all questions if possible. Do not worry if there are topics you have never covered; do your best on whatever you can attempt.

Questions are not necessarily in order of difficulty.

Marks for parts of questions are given in brackets as a guide.

Show as much working as you can. Calculators are allowed and their use expected.

There is a list of formulae given, not all of which need necessarily be used in this paper.

The paper has twenty-seven questions. Work quickly.

There are one hundred marks in total.

NAME: AGE:

PRESENT SCHOOL:

Q1.

The equation of the line L_1 is y = 3x - 2

The equation of the line L_2 is 3y - 9x + 5 = 0

Show that these two lines are parallel.

(Total for question = 2 marks)

Q2.

ABC is a right-angled triangle.



Calculate the length of *AB*. Give your answer correct to 3 significant figures.

..... cm

Q3.

Julia buys a washing machine.

20% VAT is added to the price of the washing machine. Julia then has to pay a total of $\pounds 600$

What is the price of the washing machine with **no** VAT added?

£

(Total for question = 2 marks)

Q4.

Show that (x + 1) (x + 2) (x + 3) can be written in the form $ax^3 + bx^2 + cx + d$ where *a*, *b*, *c* and *d* are positive integers.

The diagram shows triangle ${\bf A}$ drawn on a grid.



Rob reflects triangle **A** in the *x*-axis to get triangle **B**. He then reflects triangle **B** in the line y = x to get triangle **C**.

Sue reflects triangle **A** in the line y = x to get triangle **D**. She is then going to reflect triangle **D** in the *x*-axis to get triangle **E**.

Sue says that triangle **E** should be in the same position as triangle **C**.

Is Sue correct?

You must show how you get your answer.

Q6.

Here are the first five terms of a sequence.

4 11 22 37 56

Find an expression, in terms of *n*, for the *n*th term of this sequence.

.....

(Total for question = 3 marks)

Q7.

There are 10 boys and 20 girls in a class. The class has a test.

The mean mark for all the class is 60 The mean mark for the girls is 54

Work out the mean mark for the boys.

.....

Q8.

Here are some graphs.



In the table opposite, match each equation with the letter of its graph.

Equation	Graph
$y = \sin x$	
$y = x^3 + 4x$	
$y = 2^x$	
$y = \frac{4}{x}$	

(Total for question = 3 marks)

•

Q9.

Using algebra, i.e. without a calculator, prove that	$0.1\dot{3}\dot{6}\times0.\dot{2}$	is equal in value to	$\frac{1}{33}$
			33

Q10.

Richard invests £6000 for 5 years. The investment gets compound interest of $x^{0/6}$ per annum.

At the end of 5 years the investment is worth $\pounds 8029.35$

Work out the value of *x*.

.....

(Total for question = 3 marks)

Q11.

$$16^{1/5} \times 2^{x} = 8^{3/4}$$

Work out the exact value of *x*.

.....



ABC and *EDC* are straight lines. *EA* is parallel to *DB*.

EC = 8.1 cm. DC = 5.4 cm. DB = 2.6 cm.

(a) Work out the length of *AE*.

..... cm (2)

AC = 6.15 cm.

(b) Work out the length of *AB*.

..... cm (2)

Q13.

Planet	Distance from Earth (km)	Mass (kg)
Earth	0	5.97×10^{24}
Jupiter	6.29 × 10 ⁸	1.898×10^{27}
Mars	7.83×10^{7}	6.42×10^{23}
Mercury	9.17×10^7	3.302×10^{23}
Neptune	4.35 × 10°	1.024×10^{26}
Saturn	1.28×10^{9}	5.68×10^{26}
Uranus	2.72 × 10°	8.683×10^{25}
Venus	4.14×10^{7}	4.869×10^{24}

The table shows some information about eight planets.

(a) Write down the name of the planet with the greatest mass.

.....

(1)

(b) Find the difference between the mass of Venus and the mass of Mercury.

Ben says that Neptune is over a hundred times further away from Earth than Venus is.

(c) Is Ben right?

You must show how you get your answer.



ABCD is a rhombus.

The coordinates of A are (5,11).

The equation of the diagonal *DB* is $y = \frac{1}{2}x + 6$

Find an equation of the diagonal AC.

.....

Q15.

ABCD is a quadrilateral.





Prove that AC = BD.

(Total for question = 4 marks)

Q16.

$$\frac{3x-2}{4} - \frac{2x+5}{3} = \frac{1-x}{6}$$

Solve

x =

Q17.

The table shows a set of values for *x* and *y*.

x	1	2	3	4
у	9	$2\frac{1}{4}$	1	$\frac{9}{16}$

y is **inversely** proportional to the square of *x*.

(a) Find an formula for *y* in terms of *x*.

		(2)
(b)	Find the positive value of x when $y = 16$	
		(2)

(Total for question = 4 marks)

Q18.

White shapes and black shapes are used in a game. Some of the shapes are circles. All the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is 3:7

The ratio of the number of white circles to the number of white squares is 4:5

The ratio of the number of black circles to the number of black squares is 2:5

Work out what fraction of all the shapes are circles.

(Total for question = 4 marks)

.....

Q19.

The diagram shows 3 identical circles inside a rectangle.

Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram.



The radius of each circle is 24 mm.

Work out the area of the rectangle. Give your answer correct to 3 significant figures.

(Total for question = 4 marks)

Q20. There are 9 counters in a bag.

7 of the counters are green.2 of the counters are blue.

Emma takes at random two counters from the bag.

Work out the probability that Emma takes one counter of each colour. You must show your working. Q21. You are given that

$$2 - \frac{x+2}{x-3} - \frac{x-6}{x+3}$$
 can be written as a single fraction in the form $\frac{ax+b}{x^2-9}$

where *a* and *b* are integers.

Work out the value of *a* and the value of *b*.

a =

b =

(Total for question = 4 marks)

Q22.

Marc bakes 420 cakes. He bakes only vanilla cakes, banana cakes, lemon cakes and chocolate cakes.

 $\frac{2}{7}$ of the cakes are vanilla cakes.

35% of the cakes are banana cakes.

The ratio of the number of lemon cakes to the number of chocolate cakes is 4:5

Work out the number of lemon cakes Marc bakes.

.....

Q23.

Neil drove 56 km from Liverpool to Manchester. He then drove 61 km from Manchester to Sheffield.

Neil 's average speed from Liverpool to Manchester was 70 km/h. Neil took 75 minutes to drive from Manchester to Sheffield.

(a) Work out Neil's average speed for his total drive from Liverpool to Sheffield.



The area of triangle *ABC* is $6\sqrt{2}$ m^2 .

Calculate the value of *x*.

Give your answer correct to 3 significant figures.

.....



A, *B* and *C* are points on a circle of radius 5 cm, centre *O*. *DA* and *DC* are tangents to the circle. DO = 9 cm

Work out the length of arc *ABC*. Give your answer correct to 3 significant figures.

..... cm

(Total for question = 5 marks)

Q25.

Q26.

(a) Simplify
$$\frac{x^2 - 16}{2x^2 - 5x - 12}$$

(3)

(b) Make *v* the subject of the formula

$$w = \frac{15(t - 2v)}{v}$$

(3)

Q27.

Solve algebraically the simultaneous equations

$$x^2 + y^2 = 25$$

$$y - 3x = 13$$

.....

(Total for question = 5 marks)

END OF PAPER. CHECK ALL YOUR WORK

Formula Sheet



Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\sin A \quad \sin B \quad \sin b$$

Cosine Rule:
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of a triangle =
$$\frac{1}{2} ab \sin C$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$