Westminster School Challenge 2006

Mathematics III

1½ hours

You may not use a calculator in this paper.

All working should be clearly shown.

You should attempt as many questions as possible, in any order you like.

1 In a school, there are 58 pupils in year eight. Fifteen boys are in the rugby team and eleven boys are in the football team. Four boys are in both teams and nine boys are in neither team. How many girls are there in year eight?

2 There are fifteen players in a hockey squad. In one match, even though there are exactly eleven players on the pitch at any one time, all fifteen players get a game, and indeed, each member of the squad is on the pitch for the same length of time. If the match lasts an hour an a half, for how many minutes is each of them on the pitch?

3 A single railway line passes through five stations, Aytown, Beeham, Ceechester, Deebridge and Eeville, in that order. Trains run in both directions and stop at all the stations. The thin controller wants to print tickets for all the possible one-way journeys (for instance, the trip from Beeham to Deebridge and the return journey from Deebridge to Beeham will need different tickets).

a How many tickets does he have to print?

b How many tickets would he have to print if there were ***n*** stations on the line?

4 **a** **i** Calculate $\frac{7}{12} + \frac{11}{20}$.

ii Which fraction is larger, $\frac{7}{12}$ or $\frac{11}{20}$?

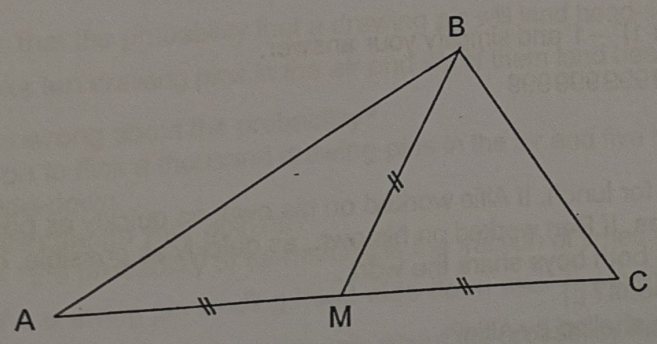
iii Find a fraction which is between $\frac{7}{12}$ and $\frac{11}{20}$ in size.

b **i** Find a fraction which is between $\frac{1}{3}$ and $\frac{2}{5}$ in size.

ii Find the fraction which is between $\frac{1}{3}$ and $\frac{2}{5}$ in size and has the smallest possible denominator. Show how you can be sure that no smaller denominator will work.

Reminder The denominator is the number on the bottom of the fraction: e.g., the denominator of $\frac{2}{5}$ is 5.

5 In the diagram, the lengths of AM and BM and CM are all equal.



a If angle BAM = 28°, find angle ABC, showing and explaining each step in your working.

b Show that, *whatever* the size of angle BAM, angle ABC is *always* a right angle.

6 Five hippopotamuses are being weighed. Adeline and Brigitte together weigh 2500 kg; Brigitte and Cecilia together weigh 3700 kg; Cecilia and Daphne together weigh 3400 kg; Daphne and Eugenie together weigh 2400 kg. All five hippopotamuses together weigh 7000 kg. What does each hippopotamus weigh?

7 Joe is a gambler.

a He plays a game which doubles his money if he wins and costs him £10 if he loses. In six rounds, he wins, loses, wins, loses, wins and finally loses. At this point he finds that he has no money left. How much did he have to start with?

b He plays another game which triples his money if he wins and costs him £10 if he loses. In six rounds, he wins, loses, wins, loses, wins and finally loses. At this point he finds that he has the same amount of money as he started with. How much was that?

c He plays a third game which pays him £30 if he wins and halves his money if he loses. In six rounds, he wins, loses, wins, loses, wins and finally loses. At this point he finds that he has twice as much money as he started with. How much was that?

8 In one year at a school there are two classes. Class A is half boys and half girls; class B is two thirds boys and one third girls. What fraction of the whole year are boys if

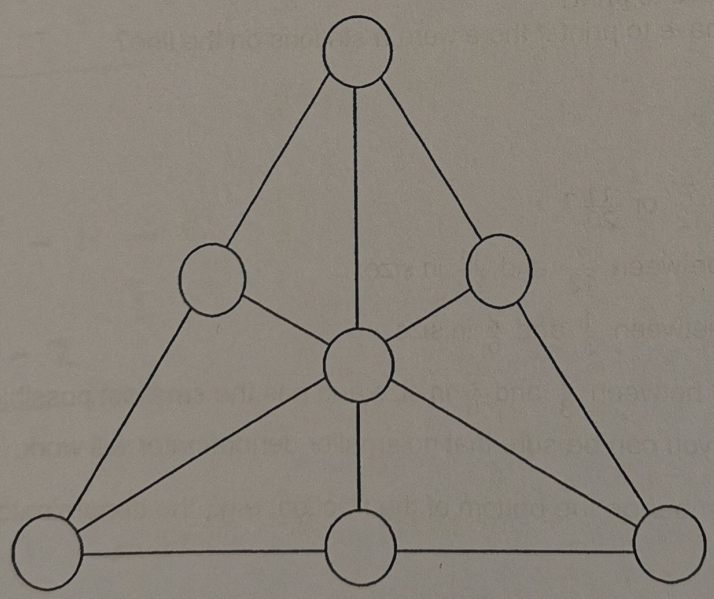
a the two classes are the same size;

b class A is twice the size of class B.

9 In the diagram shown, a number must be written into each of the seven circles, in such a way that the sum of the two numbers at the ends of each of the six straight lines in the diagram is equal to the number in the middle of that line.

a Show that the middle number is the sum of the three numbers at the corners of the triangle.

b Show that if the numbers 1 to 7 are used to complete the diagram, the seven must be in the centre of the diagram and the numbers at the corners of the triangle must be 1, 2, 4.



10 a Multiply out the brackets in $(n+1)^2 - 1$ and simplify your answer.

b Solve the equation $n(n+2) = 9999999999$

11 Alfie and Ben have to shell the peas for lunch. If Alfie worked on his own, as quickly as possible, he would take two hours to shell the peas. If Ben worked on his own, as quickly as possible, he would take three hours to shell the peas. In fact, both boys share the work.

i How quickly can they shell the peas?

ii What fraction of the peas will be shelled by Alfie?

12 The table on the right is constructed so that each entry is two thirds of the entry to its left and three fifths of the entry above it.

225	150	100
135	90	60
81	54	36

a **i** Starting on any square, move one square down and one to the right. The entry in the square you arrive at is always two fifths of the entry you started on. Explain why.

ii Starting on any square, move one square down and one to the left. The entry in the square you arrive at is always nine tenths of the entry you started on. Explain why.

b The table on the right is constructed so that each entry is a constant fraction of the entry to its left and a different constant fraction of the entry above it. Copy and complete the table.

300	120	
225		

c The table on the right is constructed so that each entry is a constant fraction of the entry to its left and a different constant fraction of the entry above it. Copy and complete the table.

900		
		480
		400

d The table on the right is constructed so that each entry is a constant fraction of the entry to its left and a different constant fraction of the entry above it.
Give two *different* ways of completing the table.

800		
	120	

Note that the two tables shown do *not* count as different as one is a reflection of the other.

225	150	100
135	90	60
81	54	36

225	135	81
150	90	54
100	60	36

e The table on the right is constructed so that each entry is a constant fraction of the entry to its left and a different constant fraction of the entry above it. Copy and complete the table.

144		
		54
	48	

13 Rufus thinks that the probability that a drawing pin will land head-side-down when it is flicked in the air is $\frac{1}{3}$. He flicks ten drawing pins in the air and six of them land head-side-down.

a Is Rufus wrong about the probability?

Rufus goes on to flick a thousand drawing pins in the air and five hundred and eighty seven of them land head-side-down.

b Does this change your opinion?

George says "the probability of something is the fraction of times it has happened. Therefore the probability of a drawing pin landing head-side-down *was* $\frac{6}{10}$; then it changed to $\frac{587}{1000}$ in the second experiment. It doesn't make sense to talk about the probability that something *will* happen because you don't know about the future."

c Do you agree with George or not? Explain your answer.