

Eton 2020 King's Scholarship B Solutions

- 1) a) 225cm^2
 b) i) $1 : 1.33$
 c) 240cm^2
- 2) a) 38.5kg
 b) 4.6kg
- 3) a) Let $\angle BCA = x$. Then $\angle BAC = 90 - x$ and $\angle ABD = x$. And have all the angles needed.
 b) 6.72cm
 c) Left square side length x . Lengths along bottom: $\frac{4}{3}x + x = 4$. $x = \frac{12}{7}$
 Right square side length y . Lengths along bottom: $\frac{4}{3}y + y + \frac{3}{4}y = 5$. $y = \frac{60}{37}$.
 Ratio follows.
- 4) a) A must be Teacher as the others say they are something else so couldn't be the truthful Teacher.
 B can't be Oppidan as then telling truth. So B is the Scholar. C is the Oppidan.
 b) A can't be the Oppidan as then would be telling the truth. A lying so Scholar.
 B must be lying so B is the Oppidan.
 C Teacher.
 c) C can't be Teacher as then would say I am teacher. B is telling truth. B is Scholar. If B were teacher then would agree with C. C Oppidan. A Teacher.
 d) If C telling truth then A Oppidan then B Teacher and C Scholar.
 If C lying then A not Oppidan (so A is Teacher or Scholar) then A could be lying or not depending on whether Teacher or Scholar.
 If A Teacher then B is Scholar and C is Oppidan.
 If A is Scholar then B is Teacher and C is Oppidan.
 So not enough information to determine who Teacher is and B lying.
 So A Teacher, B Scholar and C Oppidan.
- 5) a) $p = -4\frac{1}{2}, q = 17\frac{1}{2}, r = 19\frac{1}{2}$
 b) $x = 31.25, y = 36, z = 24$
 c) $a = 6, b = 8, c = 12$
- 6) a) 37 and 73
 b) $x=5, y=7, z=8$
- 7) a) i) $4 < 5\frac{7}{13} < 6 < 6\frac{1}{2} < 9$
 ii) $50 < 94\frac{2}{17} < 200 < 425 < 800$
 b) i) Any square number is ≥ 0 so $(a - b)^2 \geq 0$
 ii) $(a - b)^2 \geq 0$
 $a^2 - 2ab + b^2 \geq 0$
 $2ab \leq a^2 + b^2$
 c) Let $a = \sqrt{x}$ and $b = \sqrt{y}$ and substitute in previous part, then dividing by 2.
 d) In part c, divide both sides by $(x + y)$ and multiply both sides by 2 and \sqrt{xy}

- 8) a) Robin is at the start line after 10, 20, 30 minutes. After 10 and 20 the others aren't at the start, after 30 they are.
- b) After 15 minutes Robin is halfway round (1.5 laps) and Joel (12 laps) and Nick (5 laps) are at the start line.
- c) Speeds are $\frac{4}{5}, \frac{1}{3}, \frac{1}{10}$ laps per minute.

Joel and Nick are half a lap apart every s minutes, where $\frac{4}{5}s - \frac{1}{3}s = \frac{1}{2}$ so $s = \frac{15}{14}$

Nick and Robin are half a lap apart every t minutes, where $\frac{1}{3}t - \frac{1}{10}t = \frac{1}{2}$ so $t = \frac{15}{7}$

So after $\frac{15}{7}$ minutes they are all on a straight line.

Joel has done $\frac{4}{5} \times \frac{15}{7} = \frac{12}{7} = 1\frac{5}{7}$ laps.

Nick has done $\frac{1}{3} \times \frac{15}{7} = \frac{5}{7} = \frac{5}{7}$ laps.

Robin has done $\frac{1}{10} \times \frac{15}{7} = \frac{12}{7} = \frac{3}{14}$ laps.

So Joel and Nick are both opposite Robin at that point.

The time is $\frac{15}{7}$ minutes = 2 minutes and 9 seconds