

Eton 2021 King's Scholarship A Solutions

- 1) a) $28\frac{1}{3}$
 b) $18\frac{2}{3}$
 c) $599\frac{5}{36}$
 d) $10\frac{9}{16}$
- 2) a) 0.000077
 b) 7.5
 c) -1.331
- 3) a) $1\frac{1}{2}$
 b) -5
- 4) a) $8x - 15y$
 b) a
- 5) $x > -3\frac{1}{5}$
- 6) $3\frac{11}{20}$
- 7) 16 years
- 8) 131°
- 9) 65.2%
- 10) 6
- 11) a) 2 hours 55 minutes
 b) $\frac{AB}{A+B}$
 c) $\frac{3pr^2}{5q^2}$
 d) $\frac{1\frac{7}{15}yz}{x}$
- 12) a) 2:25
 b) $2\pi rh$
 c) $29\pi p^2$
- 13) a) i) $10x = 75.\dot{3}$
 $100x = 753.\dot{3}$
 ii) The difference between the equations above gives: $90x = 678$.
 Divide both sides by 90 gives $x = \frac{678}{90}$ i.e. $7.5\dot{3} = 7\frac{8}{15}$
- b) $y = 1.\dot{9}$
 $10y = 19.\dot{9}$
 $9y = 18$
 $y = 2$
- c) $10z = 17b.\dot{c}$
 $100z = 17bc.\dot{c}$
 $90z = 17bc - 17b = 1000 + 700 + 10b + c - 100 - 70 - b = 1530 + 9b + c$
 $z = \frac{1530+9b+c}{90}$
- 14) a) Triangle AMC is right angled with $AM=MC=\sqrt{2}$ so by Pythagoras $AC=2$
 b) By symmetry on DM, angle $MLC=90^\circ$. Similarly $MNC=90^\circ$. So $NML=90^\circ$.
 So CNML is a rectangle (and perhaps a square).
 $CL=CN=1$ by symmetry. So $LM=MN=1$ also (opposite sides in a rectangle).

- c) $ML=1$ (as above). $MD=\sqrt{2}$. So $DL=\sqrt{2} - 1$
- d) $DE=\sqrt{1^2 + (\sqrt{2} - 1)^2} = \sqrt{4 - 2\sqrt{2}}$
- e) $\text{Arc AE} = \sqrt{2}\pi > AB + BC + CD + DE = 4DE$
 $\sqrt{2}\pi > 4\sqrt{4 - 2\sqrt{2}}$
 $2\pi > 4\sqrt{2}\sqrt{4 - 2\sqrt{2}}$
 $2\pi > 4\sqrt{2(4 - 2\sqrt{2})}$
 $\pi > 2\sqrt{8 - 4\sqrt{2}}$
 $\pi > 2\sqrt{4(2 - \sqrt{2})}$
 $\pi > 4\sqrt{2 - \sqrt{2}}$

15)a)

Stage	Pole A Lowest position → Highest		Pole B Lowest position → Highest		Pole C Lowest position → Highest	
	Initial Position	2	1			
Move One	2		1			
Move Two			1		2	
Move Three					2	1

b)

Stage	Pole A Lowest position → Highest			Pole B Lowest position → Highest		Pole C Lowest position → Highest		
	Initial Position	3	2	1				
Move One	3	2				1		
Move Two	3			2		1		
Move Three	3			2	1			
Move Four				2	1	3		
Move Five	1			2		3		
Move Six	1					3	2	
Move Seven						3	2	1

c)

Stage	Pole A Lowest position → Highest				Pole B Lowest position → Highest			Pole C Lowest position → Highest			
	Initial Position	4	3	2	1						
Move One	4	3	2		1						
Move Two	4	3			1			2			
Move Three	4	3						2	1		
Move Four	4				3			2	1		
Move Five	4	1			3			2			
Move Six	4	1			3	2					
Move Seven	4				3	2	1				
Move Eight					3	2	1	4			
Move Nine					3	2		4	1		
Move Ten	2				3			4	1		
Move Eleven	2	1			3			4			
Move Twelve	2	1						4	3		
Move Thirteen	2				1			4	3		
Move Fourteen					1			4	3	2	
Move Fifteen								4	3	2	1

- d) $2^n - 1$. When we have an extra disc, we have to shift all those above ('small pile') to another pole ($2^{n-1} - 1$), then shift the bottom disc and then shift the 'small pile' back on top of the biggest disc ($2^{n-1} - 1$).
 $(2^{n-1} - 1) + (2^{n-1} - 1) + 1 = 2^n - 1$